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THE EFFECT OF INITIAL IMPERFECTIONS ON
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by

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This report deals with the effect of initial imperfections on optimal stiffened cylinders under axial compression. A nonlinear stability methodology is developed for analyzing such system in the presence of small initial imperfections. This methodology is then employed to assess the effectiveness of stiffened cylinders optimized on the basis of linear stability analysis. This research was performed under Grant AFOSR 74-2655 during the period 1 Feb 76 through 31 Jan 77.

ABSTRACT

The buckling analysis of imperfect, thin, circular, cylindrical, stiffened shell under uniform axial compression, for various boundary conditions is first investigated. A methodology is presented for predicting critical conditions for such configurations. This methodology is based on the smeared technique and the von Kármán-Donnell nonlinear kinematic relations in the presence of geometric imperfections. The computational procedure employs a Fourier series type of separated solution and through the Galerkin procedure the field equations are reduced to a system of ordinary differential equations. These equations are solved by the finite difference scheme. Numerical results for numerous stiffened and unstiffened configurations are presented.

Then the effect of initial geometric imperfections on the optimal stiffened circular cylindrical shell sunder uniform axial compression is assessed. The imperfection sensitivity of geometries corresponding to values of the design variables surrounding the optimal configuration is investigated for two design configurations. A design methodology is proposed through which one may arrive at the minimum weight configuration in the presence of geometric imperfections of predetermined maximum amplitude and of a shape that yields the greatest reduction in the linear theory buckling load.

TABLE OF CONTENTS

NOTATIONS

A	Area
A_x, A_y	Stringer and ring cross-sectional area
D	Flexural stiffness of the skin
E	Young's modulus of elasticity
E_{xx_p}	Extensional stiffness of the skin
e_x, e_y	Stringer and ring eccentricities (positive inward)
e	Unit end shortening
F	Stress function
f_i	Fourier coefficient of stress function
I_{xc}, I_{yc}	Stringer-and ring moment of inertia about their centroidal axes
K	Number of terms in truncated Fourier Series
ℓ_x, ℓ_y	Stringer and ring spacings
ℓ	Mesh point
L	Total length of the shell
M_{xx}, M_{yy}, M_{xy}	Moment resultants
m	Number of axial half waves
N_{xx}, N_{yy}, N_{xy}	Stress resultants
$\bar{N}_{xx}$	Applied compressive load
$N_{x_{c\ell}}$	Classical buckling load
$N_{x_{cr}}$	Critical load (limit point)
n	Number of circumferential full waves

NP	Number of points in axial direction
Q^*	Effective transverse shear
R	Radius of the cylinder
t	Skin thickness
U_T	Total Potential
u,v	In-plane displacements
w	Radial displacement (positive inward)
w^o	Radial geometric imperfection
x,y,z	Coordinate system
Z	Batdorf curvature parameter [$= L^2(1-\nu^2)^{1/2}/R.t$]
z	Unknowns vector
$\epsilon_{xx},\epsilon_{yy},\epsilon_{xy}$	Reference surface strains
$\varkappa_{xx},\varkappa_{yy},\varkappa_{xy}$	Reference surface changes in curvature and torsion
$\lambda_{xx},\lambda_{yy}$	Smeared extensional stiffnesses of stringers and rings
ν	Poisson's ratio
ρ_{xx},ρ_{yy}	Smeared flexural stiffnesses of stringers and rings
Δ	Interval size between mesh point
$(\ )' = [\ ]_{,x}$	Derivative with respect to x

CHAPTER I

INTRODUCTION

Stability of thin circular cylindrical shells (with or without stiffening), because of its importance, has enjoyed tremendous attention for the past seventy years. Although a complete understanding of all the details of the phenomena involved has not yet been reached, it has been well established that the discrepancy between classical theoretical predictions and experimental results lies primarily in the fact that the system is sensitive to geometric imperfections, the presence of which is unavoidable.

The imperfection sensitivity of the system was initially established through strict postbuckling analyses of the perfect geometry system. In addition, it was explained that, the load carrying capacity of such systems is directly related to the lowest load corresponding to postbuckling states of equilibrium. The first theoretical investigation of this type was reported by von Kármán and Tsien[1] in 1941. The investigators calculated postbuckling equilibrium states, for an unstiffened, axially compressed, thin, circular cylindrical shell, corresponding to loads far below the classical critical load. The calculations were based on a number of simplifying assumptions the most important being the neglect of the effect of the boundary conditions. Many subsequent investigators[2-4] attempted to improve the calculations of von Kármán and Tsien in order to find the smallest postbuckling equilibrium load. This search came to an end when Hoff, Madsen and Mayers[5] found in their calculations that the

minimal postbuckling load tended towards zero with improved functional representation (taking more and more terms in the series) for the solution to the governing equations and with diminishing thickness. In addition, Madsen and Hoff[6] repeated the investigation by employing more accurate kinematic relations than those of Donnell. The difference between these results and the previous ones was insignificant. Futhermore, Koiter[7] has shown that the von Kármán-Donnell equations are also applicable to problems with arbitrarily large displacements, when the function describing the radial displacement is taken to be the curvature function defined in his paper.

Koiter[8] was the first to question the use of the minimal postbuckling equilibrium load as a measure of the load carrying capacity of the configuration. Instead, he proposes to find the critical load (limit point) of the imperfect system. Koiter's work is the first attempt to the buckling analysis of an imperfect shell, but his method is limited to the neighborhood of the classical load and therefore to small imperfections of certain spatial form. This approach has been adopted by many investigators including Hutchinson and Amazigo[9] who treated the stringer or ring stiffened thin cylindrical shell. Excellent reviews on the subject may be found in the works of Hoff[10], and Hutchinson and Koiter[11].

Many of the postbuckling analyses that are based on Koiter's proposition (see Ref. 11) disregard the effect of end conditions by assuming that the cylinder length is extremely large. A systematic experimental investigation dealing with cylinders of various lengths (Ref. 12) revealed that

the postbuckling behavior is strongly influenced by the cylinder length. Narasinham and Hoff[13,14] analyzed an unstiffened, thin, circular, cylindrical, imperfect shell of finite length under uniform axial compression. They solve the nonlinear equations by employing a separated series solution for the dependent variables, each term of which contains a function of x multiplied by a cosine term in y (Fourier). Thus the equations are reduced to a system of ordinary differential equations, which in turn are solved by the finite difference scheme. Although the equations are developed for arbitrary terms of Fourier series, the solution is restricted to just one term for the displacement function. A similar procedure, but one that employs the "shooting method" (Ref. 15) instead of the finite difference technique, is employed by Arbocz and Sechler[16] in their investigation of the buckling behavior of axially compressed imperfect cylindrical shells.

With the exception of the work of Ref. 9, there is virtually no reported investigation on the buckling behavior of imperfect stiffened configurations. The first part of the report presents a methodology for analyzing the buckling of a uniformly compressed, stiffened (rings and stringers), thin, circular, cylindrical imperfect shells of finite length and various boundary conditions. The analysis employs the von Kármán-Donnell large displacement equations and the smeared technique. The solution procedure is similar to that of Ref. 13 but the limitations on the spatial character of the imperfection has been relaxed considerably. Results have been produced for special case geometries that have been reported in the open literature (bench marks) and for new configurations

of the stiffened type in both directions.

The latter part of the present report applies this method to the investigation of the effect of initial geometric imperfections on the optimal (linear theory) stiffened cylinder configuration under uniform axial compression.

CHAPTER II

MATHEMATICAL FORMULATION OF THE NONLINEAR BUCKLING ANALYSIS

By employing the von Kármán-Donnell kinematic equations for geometrically imperfect [$w^o(x,y)$] thin cylindrical shells one can easily derive the compatibility and transverse equilibrium equations in terms of the radial displacement w and the stress function F, as well as the expressions for the total potential and the "unit end shortening". The procedure employed is similar to that outlined in Ref. 13 for unstiffened, imperfect thin cylindrical shells.

Consider a geometrically imperfect stiffened cylinder under uniform axial compression. Let $w^o(x,y)$ denote the deviation of the shell mid-surface (taken to be the reference surface) from the corresponding perfectly cylindrical one. Let u,v, and w denote the displacements of material points on the reference surface (see Fig. 1).

The kinematic relations, first proposed by Donnell[17] are given below

$$\begin{aligned}
\epsilon_{xx} &= u,_x + \tfrac{1}{2}(w,_x^2 + 2w,_x w^o,_x) \\
\epsilon_{yy} &= v,_y - w/R + \tfrac{1}{2}(w,_y^2 + 2w,_y w^o,_y) \\
\gamma_{xy} &= 2\epsilon_{xy} = u,_y + v,_x + w,_x w,_y + w,_y w^o,_x + w,_x w^o,_y \\
\kappa_{xx} &= w,_{xx}; \quad \kappa_{yy} = w,_{yy}; \quad \kappa_{xy} = w,_{xy}
\end{aligned} \tag{1}$$

The stress and moment resultants to strains and changes in curvature and torsion are taken from Ref. 18. They were derived by employing the smeared technique.

$$N_{xx} = E_{xx_p}\left[(1+\lambda_{xx})\epsilon_{xx} + \nu\epsilon_{yy} - e_x\lambda_{xx}\varkappa_{xx}\right]$$

$$N_{yy} = E_{xx_p}\left[\nu\epsilon_{xx} + (1+\lambda_{yy})\epsilon_{yy} - e_y\lambda_{yy}\varkappa_{yy}\right]$$

$$N_{xy} = E_{xx_p}\left[1-\nu)\epsilon_{xy}\right]$$

$$M_{xx} = D\left\{\left[(1+\rho_{xx}) + \frac{12}{t^2} e_x^2\lambda_{xx}\right]\varkappa_{xx} + \nu\varkappa_{yy} - \frac{12}{t^2} e_x\lambda_{xx}\epsilon_{xx}\right\} \tag{2}$$

$$M_{yy} = D\left\{\nu\varkappa_{xx} + \left[(1+\rho_{yy}) + \frac{12}{t^2} e_y^2\lambda_{yy}\right]\varkappa_{yy} - \frac{12}{t^2} e_y\lambda_{yy}\epsilon_{yy}\right\}$$

$$M_{xy} = D(1-\nu)\,\varkappa_{xy}$$

where

$$E_{xx_p} = Et/(1-\nu^2);\quad D = Et^3/12(1-\nu^2);\quad \lambda_{xx} = A_x(1-\nu^2)/t\ell_x$$

$$\lambda_{yy} = A_y(1-\nu^2)/t\ell_y;\quad \rho_{xx} = EI_{xc}/D\ell_x;\quad \text{and } \rho_{yy} = EI_{yc}/D\ell_y.$$

From Eqs. (2) one may derive the following expressions for the reference surface strains

$$\epsilon_{xx} = a_1N_{xx} + a_2N_{yy} + a_3\varkappa_{xx} + a_4\varkappa_{yy}$$

$$\epsilon_{yy} = a_2N_{xx} + b_2N_{yy} + b_3\varkappa_{xx} + b_4\varkappa_{yy} \tag{3}$$

$$\epsilon_{xy} = \tfrac{1}{2}\gamma_{xy} = N_{xy}/(1-\nu)E_{xx_p}$$

where

$$a_1 = (1+\lambda_{yy})/\alpha E_{xx_p};\quad a_2 = -\nu/\alpha E_{xx_p};\quad a_3 = (1+\lambda_{yy})e_x\lambda_{xx}/\alpha$$

$$a_4 = -\nu e_y\lambda_{yy}/\alpha;\quad b_2 = (1+\lambda_{xx})\alpha E_{xx};\quad b_3 = -\nu e_x\lambda_{xx}/\alpha \tag{4}$$

$$b_4 = (1+\lambda_{xx})e_y\lambda_{yy}/\alpha; \quad \alpha = [1+\lambda_{xx})(1+\lambda_{yy}) - \nu^2]$$

By employing the principle of the stationary value of the total potential one can derive the following equilibrium equations

$$N_{xx,x} + N_{xy,y} = 0$$

$$N_{xy,x} + N_{yy,y} = 0$$

$$M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} = \frac{N_{yy}}{R} + [N_{yy}(w,_y + w^o,_y)],_y + [N_{xy}(w,_x + w^o,_x)],_y$$
$$+ [N_{xx}(w,_x + w^o,_x)],_x + [N_{xy}(w,_y + w^o,_y)],_x$$

By introducing the Airy stress function, as $N_{xx} = -\bar{N}_{xx} + F,_{yy}$, $N_{yy} = F,_{xx}$ and $N_{xy} = -F,_{xy}$ where $\bar{N}_{xx}$ is the level of the applied uniform axial compression, the first two of Eqs. (5) are identically satisfied.

Next, by eliminating u and v from the first three of Eqs. (1), employing Eqs. (3), the Airy stress function and the last three of Eqs. (1) one can derive the compatibility equation in terms of the Airy stress function, F and the radial displacement, w. If one expresses the third of Eq. (5) in terms of F and w, the governing equations consist of two coupled partial differential equations in F and w. These are:

Equilibrium

$$DL_h[w] - L_q[F] - F,_{xx}/R + \bar{N}_{xx}(w,_{xx} + w^o,_{xx}) - L[F, w + w^o] = 0 \qquad (6)$$

Compatibility

$$L_d[F] + L_q[w] + \tfrac{1}{2}L[w, w + 2w^o] + w,_{xx}/R = 0 \qquad (7)$$

where L_d, L_h, and L_q are differential operators defined by L_g,

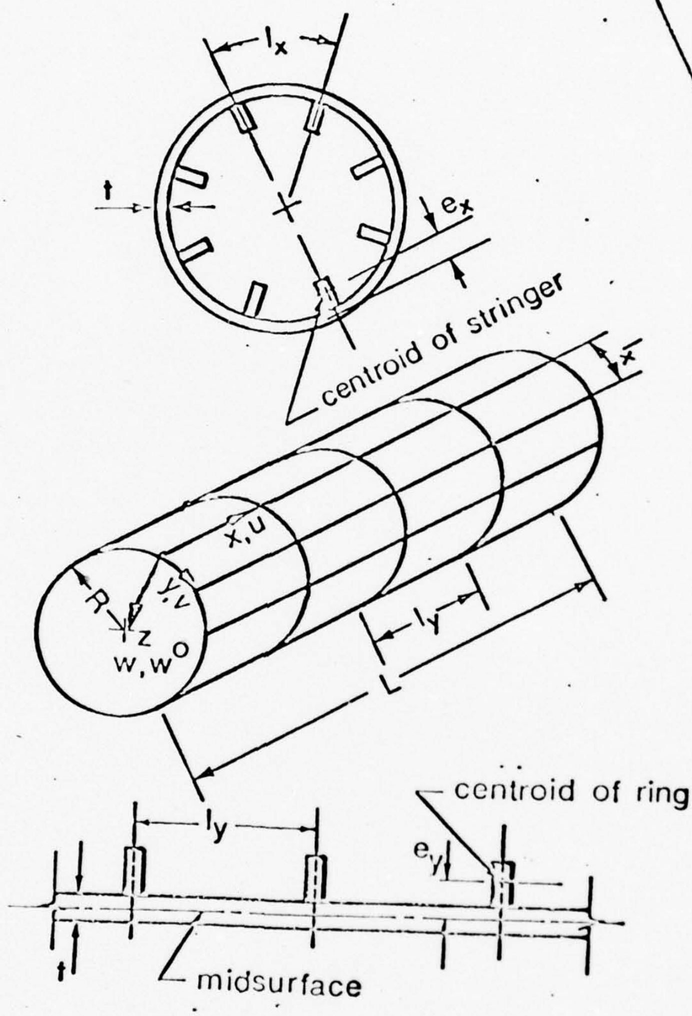


Fig. 1. Geometry and Sign Convention

$$L_g[s] = g_{11}s,_{xxxx} + 2g_{12}s,_{xxyy} + g_{22}S,_{yyyy} \tag{8a}$$

with

$$d_{11} = (1+\lambda_{xx})/\alpha E_{xx_p}$$

$$d_{12} = [(1+\lambda_{xx})(1+\lambda_{yy}) - \nu]/\alpha(1-\nu)E_{xx_p} \tag{9a}$$

$$d_{22} = (1+\lambda_{yy})/\alpha E_{xx_p}$$

$$h_{11} = 1 + \rho_{xx} + \frac{12}{t^2} \cdot \frac{e_x^2\lambda_{xx}(1 + \lambda_{yy} - \nu^2)}{\alpha}$$

$$h_{12} = 1 + \frac{12}{t^2} \frac{\nu e_x e_y \lambda_{xx}\lambda_{yy}}{\alpha} \tag{9b}$$

$$h_{22} = 1 + \rho_{yy} + \frac{12}{t^2} \frac{e_y^2\lambda_{yy}(1 + \lambda_{xx} - \nu^2)}{\alpha}$$

$$q_{11} = - \nu e_x\lambda_{xx}/\alpha$$

$$q_{12} = [(1 + \lambda_{yy})e_x\lambda_{xx} + (1 + \lambda_{xx})e_y\lambda_{yy}]/(2\alpha) \tag{9c}$$

$$q_{22} = - \nu e_y\lambda_{yy}/\alpha$$

and L is a differential operator defined by

$$L[S,T] = S,_{xx}T,_{yy} - 2S,_{xy}T,_{xy} + S,_{yy}T,_{xx} \tag{8b}$$

The total potential expression, in terms of the Airy stress function and the radial displacement, is given below

$$U_T = \frac{1}{2E_{xx_p}} \int_A (\beta_1 F,^2_{yy} + \beta_2 F,^2_{xx} + \beta_3 F,_{xx}F,_{yy} + \beta_4 F,^2_{xy})dA$$

$$+ \frac{D}{2} \int_A (\alpha_1 w,_{yy}^2 + \alpha_2 w,_{xx}^2 + \alpha_3 w,_{xx} w,_{yy} + \alpha_4 w,_{xy}^2) dA \tag{10}$$

$$- \frac{\bar{N}_{xx}}{2E_{xx_p}} \int_A (2\beta_1 F,_{yy} + \beta_3 F,_{xx}) dA + \frac{\beta_1}{E_{xx_p}} \pi RL \bar{N}_{xx}^2 - \bar{N}_{xx} 2\pi RL e_{AV}$$

where e_{AV} (average end shortening) is given by

$$e_{AV} = - \int_A u,_x \, dA/2\pi RL$$

and

$$\beta_1 = d_{22} E_{xx_p}; \quad \beta_2 = d_{11} E_{xx_p}; \quad \beta_3 = - 2\nu/\alpha; \quad \beta_4 = 2/(1 - \nu)$$

$$\alpha_1 = h_{22}; \quad \alpha_2 = h_{11}; \quad \alpha_3 = 2\nu\left[1 + \frac{12}{t^2} \frac{e_x e_y \lambda_{xx} \lambda_{yy}}{\alpha}\right]; \quad \alpha_4 = 2(1 - \nu) \tag{11}$$

Similarly, the expressions for the average end shortening and "unit end shortening" at y = 0 are given by

$$e_{AV} = a_1 \bar{N}_{xx} - \frac{1}{2\pi RL} \int_0^{2\pi R} \int_0^L [a_1 F,_{yy} + a_2 F,_{xx} + a_3 w,_{xx} + a_4 w,_{yy}$$

$$- \frac{1}{2} w,_x (w,_x + 2w^o,_x)] dxdy \tag{12a}$$

$$e = a_1 \bar{N}_{xx} - \frac{1}{L} \int_o^L \left[a_1 F,_{yy} + a_2 F,_{xx} + a_3 w,_{xx} + a_4 w,_{yy}\right.$$

$$\left. - \frac{1}{2} w,_x (w,_x + 2w^o,_x) \right]_{y=o} dx \tag{12b}$$

Note that e measures the amount of end shortening per unit of cylinder length, L. The associated boundary conditions are either kinematic or natural (but not both) except for the direction associated with the length

of the cylinder (x) in which case the displacement component u is free and the stress resultant N_{xx} must equal to the applied stress resultant, $-\bar{N}_{xx}$. Thus at a boundary characterized by x = 0 or x = L the boundary conditions are:

Either	or
in-plane	
$N_{xx} = F,_{yy} - \bar{N}_{xx} = -\bar{N}_{xx}$	$u = \text{constant}$
$N_{xy} = 0$	$v = 0$
transverse	
$M_{xx} = 0$	$w,_x = 0$
$Q^*_x = 0$	$w = 0$

The expressions for the moment resultant, M_{xx}, and the effective transverse shear, Q^*_x, are

$$
\begin{aligned}
M_{xx} &= \gamma_1 w,_{xx} + \gamma_2 w,_{yy} + \gamma_3 (F,_{yy} - \bar{N}_{xx}) + \gamma_4 F,_{xx} \\
Q^*_x &= (F,_{yy} - \bar{N}_{xx})(w,_x + w^o,_x) + F,_{xy}(w,_y + w^o,_y) - M_{xx,x} - 2M_{xy,y}
\end{aligned}
\tag{14}
$$

where

$$\gamma_1 = Dh_{11}; \quad \gamma_2 = D\frac{\alpha_3}{2}; \quad \gamma_3 = -\alpha_3; \quad \gamma_4 = -b_3$$

The general computer program is written for the following end conditions (SSi, CCi,FFi, i = 1,2,3,4)

SS: $w = M_{xx} = 0$

CC: $w = w,_x = 0$

FF: $Q_x^* = M_{xx} = 0$

1. $F,_{xy} = F,_{yy} = 0$
2. $F,_{xy} = 0; \quad u = C$
3. $v = F,_{yy} = 0$ (15)
4. $v = 0, \quad u = C$

where C is a constant.

The conditions in u and v can be expressed in terms of w and F as in Ref. 14. For example, the condition u = C in SS -2 can be replaced by a condition expressed solely in terms of w, w^o, F and their gradients. This is accomplished by the following procedure:

This boundary condition, SS=2, at x = 0 or L given w = 0; $F,_{xy} = 0$, $M_{xx} = 0$ and u = C. The first two are in terms of w and F. The third one, $M_{xx} = 0$, from the first of Eqs. (14) is expressed in terms of w,F, and their gradients. For the last one, one notes that [see Eqs. (1) and (3)]

$$\epsilon_{xy} = \frac{1}{2}\left[u,_y + v,_x + w,_x w,_y + w,_y w^o,_x + w,_x w^o,_y\right] = -F,_{xy}/(1-\nu)E_{xx_p} \tag{16}$$

since $F,_{xy} = 0$, $w,_y = 0$ because $w(0,y) = 0$, and $u,_y = 0$ because $u(0,y) = C$, Eq. (16) becomes

$$v,_x + w,_x w^o,_y = 0 \tag{17}$$

Similarly, from Eqs. (1) and (3) one may write

$$\begin{aligned} \epsilon_{yy} &= v,_y + \tfrac{1}{2}[w,_y(w,_y + 2w^o,_y)] - \frac{w}{R} \\ &= a_2 N_{xx} + b_2 N_{yy} + b_3 k_{xx} + b_4 k_{yy} \end{aligned} \tag{18}$$

This equation, Eq. (18), is valid at any point along the shell, there-

fore differentiation with respect to x does not violate its validity. If this is done and if the N's and k's are expressed in terms of w, F, and their gradients, one may write

$$v,_{yx} + \frac{1}{2}\left[w,_{xy}\,(w,_y + 2w^o,_y) + w,_y\,(w,_{xy} + 2w^o,_{xy})\right] - \frac{w,_x}{R}$$
$$= a_2F,_{yyx} + b_2F,_{xxx} + b_3w,_{xxx} + b_4w,_{yyx} \tag{19}$$

Evaluation of Eq. (19) at x = 0 or L, and use of the fact that $w,_y\,(0,y) = 0$ yields

$$v,_{yx} + w,_{xy}w^o,_y - w,_x/_R = a_2F,_{yyx} + b_2F,_{xxy} + b_3w,_{xxx} + b_4w,_{yyx} \tag{20}$$

Differentiation of Eq. (17) with respect to y, yields

$$v,_{xy} + w,_{xy}w^o,_y + w,_xw^o,_{yy} = 0 \tag{21}$$

Substitution of Eq. (21) into Eq. (20) yields a boundary condition equivalent to u = C, or

$$b_2F,_{xxx} + b_3w,_{xxx} + b_4w,_{yyx} + w,_x(\frac{1}{R} + w^o,_{yy}) = 0 \tag{22}$$

Similar steps may be followed to express all possible boundary conditions in terms of w, F, and their gradients. In order to save space only the final expression for all possible boundary conditions, Eqs. (15), are given below, which have been incorporated into the computer program (see Appendix B)

<u>SS-1</u> $w = \gamma_1w,_{xx} + \gamma_4F,_{xx} = F,_{xy} = F,_{yy} = 0$

<u>SS-2</u> $w = \gamma_1w,_{xx} + \gamma_3(F,_{yy} - \bar{N}_{xx}) + \gamma_4F,_{xx} = F,_{xy} = 0$

$$b_2F,_{xxx} + b_3w,_{xxx} + b_4w,_{yyx} + w,_x\left(\frac{1}{R} + w^o,_{yy}\right) = 0$$

$$\underline{\text{SS-3}} \quad w = \gamma_1 w,_{xx} + \gamma_4 F,_{xx} = F,_{yy} = b_2 F,_{xx} + b_3 w,_{xx} = 0$$

$$\underline{\text{SS-4}} \quad w = \gamma_1 w,_{xx} + \gamma_3 (F_{yy} - \bar{N}_{xx}) + \gamma_4 F,_{xx} = a_2 (F,_{yy} - \bar{N}_{xx}) + b_2 F,_{xx}$$

$$+ b_3 w,_{xx} = 0$$

$$\left[a_2 + 2/(1-\nu)E_{xx_p} \right] F,_{xyy} + b_2 F,_{xxx} + b_3 w,_{xxx} + b_4 w,_{xyy}$$

$$+ w,_x \left(\frac{1}{R} + w^o,_{yy}\right) = 0 \qquad (23a)$$

$$\underline{\text{CC-1}} \quad w = w,_x = F,_{xy} = F,_{yy} = 0$$

$$\underline{\text{CC-2}} \quad w = w,_x = F,_{xy} = 0 \qquad b_2 F,_{xxx} + b_3 w,_{xxx} = 0$$

$$\underline{\text{CC-3}} \quad w = w,_x = F,_{yy} = 0 \qquad b_2 F,_{xx} + b_3 w,_{xx} = 0$$

$$\underline{\text{CC-4}} \quad w = w,_x = a_2 \left(F,_{yy} - \bar{N}_{xx} \right) + b_2 F,_{xx} + b_3 w,_{xx} = 0$$

$$\left[a_2 + 2/(1-\nu)E_{xx_p} \right] F,_{yyx} + b_2 F,_{xxx} + b_3 w,_{xxx} = 0 \qquad (23b)$$

Similarly the FF-1 condition and the symmetry and antisymmetry conditions at $x = L/2$ are

$$\underline{\text{FF-1}} \quad \gamma_1 w,_{xx} + \gamma_2 w,_{yy} + \gamma_4 F,_{xx} = F,_{yy} = F,_{xy} = 0$$

$$\gamma_4 F,_{xxx} + \gamma_1 w,_{xxx} + \left[\gamma_2 + 2D(1-\nu) \right] w,_{xyy} + \bar{N}_{xx} (w,_x - w^o,_x) = 0$$

(23c)

<u>Symmetry</u> $(w,_x = Q^* = N_{xy} = u = 0)$

$$w,_x = \gamma_1 w,_{xxx} + \gamma_4 F,_{xxx} - (F,_{yy} - \bar{N}_{xx})\, w^o,_x = 0$$

$$F,_{xy} = b_2 F,_{xxx} + b_3 w,_{xxx} + w^o,_x w,_{yy} = 0$$

(23d)

<u>Antisymmetry</u> ($w = M_{xx} = v = F,_{yy} = 0$)

$$w = \gamma_1 w,_{xx} + \gamma_4 F,_{xx} = 0$$

$$F,_{yy} = b_2 F,_{xx} + b_3 w,_{xx} = 0$$

The problem, as formulated herein, is to find the limit point which represents the buckling load for the imperfect configuration. This implies to solve the field equations, Eqs. (6) and (7), subject to the proper boundary conditions for a given imperfection and level of the applied load (small initially), $-\bar{N}_{xx}$, and thus obtain the corresponding amount of "unit end shortening", Eq. (12b). By plotting $\bar{N}_{xx}$ versus e one can obtain the limit point (theoretically).

CHAPTER III

SOLUTION METHODOLOGY; NONLINEAR BUCKLING ANALYSIS

By employing the von Kármán-Donnell kinematic relations the field equations consist of two coupled, nonlinear, partial differential equations in terms of the transverse displacement, w, and the Airy stress function, F. The procedure employed herein for accomplishing a solution is basically similar to that of Refs. 13 and 14. The system of partial differential equations is reduced to a system of ordinary differential equations by using a separated solution (Fourier series) of the following form (see Refs. 13 and 14).

$$w(x,y) = \sum_{i=0}^{K} W_i(x) \cos \frac{iny}{R}$$

$$F(x,y) = \sum_{i=0}^{2K} f_i(x) \cos \frac{iny}{R} \qquad (24)$$

$$w^o(x,y) = \sum_{i=0}^{K} W_i^o(x) \cos \frac{iny}{R}$$

Note that $W_i^o(x)$ denotes the known coefficient of the ith component of the geometric imperfection, and n is the parameter associated with the number of full waves around the circumference.

By substituting Eqs. (24) into Eq. (7), employing trigonometric identities of double Fourier series as in Ref. 19 involving products and the orthogonality of the trigonometric functions, the compatibility equation becomes

for i = 0

$$f_o'' = \frac{1}{d_{11}}\left[-\, q_{11}W_o'' - W^o/R + \frac{n^2}{4R^2}\sum_{j=1}^{K} j^2(W_j + 2W_j^o)W_j\right] \tag{25a}$$

for i = 1,2,...2K

$$\begin{aligned}
& d_{11}f_i'''' - 2\left(\frac{in}{R}\right)^2 d_{12}f_i'' + \left(\frac{in}{R}\right)^4 d_{22}f_i \\
& \quad + \delta_i\left[q_{11}W_i'''' - 2\left(\frac{in}{R}\right)^2 q_{12}W_i'' + \left(\frac{in}{R}\right)^4 q_{22}W_i + W_i''/R\right] \\
& \quad - \frac{n^2}{4R^2}\sum_{j=0}^{K}\Big\{\Big[(i+j)^2\delta_{i+j}(W_{i+j} + 2W_{i+j}^o) \\
& \qquad + (2 - \eta_{j-i}^2)(i-j)^2\delta_{|i-j|}(W_{|i-j|} + 2W_{|i-j|}^o)\Big]W_j'' \\
& \quad + \left[\delta_{i+j}(W_{i+j}'' + 2W_{i+j}^{o''}) + (2 - \eta_{j-i}^2)\delta_{|i-j|}(W_{|i-j|}'' + 2W_{|i-j|}^{o''})\right]j^2W_j \\
& \quad + 2\left[(i+j)\delta_{i+j}(W_{i+j}' + 2W_{i+j}^{o'}) - \eta_{i-j}|i-j|\delta_{|i-j|}(W_{|i-j|}' + 2W_{|i-j|}^{o'})\right]jW_j'\Big\} = 0
\end{aligned} \tag{25b}$$

where

$$\delta_\ell = \begin{cases} 0 & \ell > K \\ 1 & \ell \le K \end{cases} \qquad \eta_\ell = \begin{cases} -1 & \ell < 0 \\ 0 & \ell = 0 \\ 1 & \ell > 0 \end{cases}$$

and

$$(\ )' = \frac{d}{dx}\ .$$

Next, if Eqs. (24) are substituted into the equilibrium equation, Eq. (6),

and the Galerkin procedure is employed ($\cos \frac{iny}{R}$ is the weighting function $i = 1,2,\ldots K$), the vanishing of the (K+1) Galerkin integrals leads to the following system of (K+1) ordinary differential equations

for i = 0

$$W_o'''' [Dh_{11} + q_{11}^2/d_{11}] + W_o''[2q_{11}/R \cdot d_{11}] + W_o[1/R^2 \cdot d_{11}] + \bar{N}_{xx}(W'' + W_o^{o}{}'')$$

$$- \frac{n^2}{4R^2} \sum_{j=1}^{K} j^2 \Big\{ \frac{q_{11}}{d_{11}} [(W_j + 2W_j^o) W_j'' + (W_j'' + 2W_j^o{}'') W_j$$

$$+ 2(W_j' + 2W_j^o{}') W_j'] + \frac{1}{Rd_{11}} [(W_j + 2W_j^o) W_j] - 2[(W_j + W_j^o) f_j''$$

$$+ (W_j'' + W_j^o{}'') f_j + 2(W_j' + W_j^o{}') f_j'] \Big\} = 0 \qquad (26a)$$

for i = i,2,...,K

$$D\left[h_{11}W_i'''' - 2\left(\frac{in}{R}\right)^2 h_{12}W_i'' + \left(\frac{in}{R}\right)^4 h_{22}W_i \right]$$

$$- \left[q_{11}f_i'''' - 2\left(\frac{in}{R}\right)^2 q_{12}f_i'' + \left(\frac{in}{R}\right)^4 q_{22}f_i + f_i''/R \right]$$

$$+ \bar{N}_{xx}(W_i'' + W_i^o{}'') - W_\alpha''\left[\frac{q_{11}}{d_{11}} \left(\frac{in}{R}\right)^2 \right] (W_i + W_i^o) - W_o\left[\frac{1}{Rd_{11}} \left(\frac{in}{R}\right)^2 (W_i \right.$$

$$\left. + W_i^o) + \frac{n^4}{4R^4} \frac{i^2}{d_{11}} (W_i + W_i^o) \sum_{j=1}^{K} j^2 (W_j + 2W_j^o) W_j \right.$$

$$+ \frac{n^2}{2R^2} \sum_{j=1}^{2K} \Big\{ [(i+j)^2 \delta_{i+j} (W_{i+j} + W_{i+j}^o)$$

$$+ (2 - \eta_{j-i}^2)\,(i-j)^2 \delta_{|i-j|} (W_{|i-j|} + W^o_{|i-j|})]f''_j$$

$$+ [\delta_{i+j}(W''_{i+j} + W^{o''}_{i+j}) + (2 - \eta_{j-i}^2)\delta_{|i-j|}(W''_{|i-j|} + W^{o''}_{|i-j|})]j^2 f_j$$

$$+ 2[\,(i+j)\delta_{i+j}\,(W'_{i+j} + W^{o'}_{i+j})$$

$$- \eta_{i-j}|i-j|\delta_{|i-j|}(W'_{|i-j|} + W^{o'}_{|i-j|})]\;jf'_j\} \;=\; 0 \qquad (26b)$$

For a given value of the applied load, $\bar{N}_{xx}$, and imperfection, Eqs. (25) and (26) represent a system of (3K + 2) coupled nonlinear differential equations in (3K + 2) unknowns, f_i $i = 0,1,2,\ldots.2K$ and W_i $i = 0,1,2,\ldots,K$. These equations denote equilibrium and compatibility conditions. Similarly, the expressions for the total potential, U_T, average end shortening, e_{AV}, and "unit end shortening", e, become

$$U_T = \pi R \int_o^L \left\langle \frac{1}{E_{xx_p}} \left\{ \frac{\beta_2}{d_{11}^2} \left[- \frac{W_o}{R} - q_{11} W''_o + \frac{n^2}{4R^2} \sum_{i=1}^{K} i^2 (W_i + 2W_i^o) W_i \right]^2 \right.\right.$$

$$\left. + \frac{1}{2} \sum_{i=1}^{2K} \left[\beta_1 \left(\frac{in}{R}\right)^4 f_i^2 + \beta_2 f_i''^2 - \beta_3 \left(\frac{in}{R}\right)^2 f''_i f_i + \beta_4 \left(\frac{in}{R}\right)^2 f_i'^2 \right] \right\}$$

$$+ D \left\{ \alpha_2 W_o''^2 + \frac{1}{2} \sum_{i=1}^{K} \left[\alpha_1 \left(\frac{in}{R}\right)^4 W_i^2 + W_i''^2 - \alpha_3 \left(\frac{in}{R}\right)^2 W''_i W_i + \alpha_4 \left(\frac{in}{R}\right)^2 W_i'^2 \right] \right\}$$

$$- \frac{\bar{N}_{xx}\beta_3}{d_{11}E_{xxp}} \left[- \frac{W_o}{R} - q_{11}W_o'' + \frac{n^2}{4R^2} \sum_{i=1}^{K} i^2 (W_i + 2W_i^o)W_i \right] \Bigg\rangle dx$$

$$+ \frac{\beta_1 \pi RL}{E_{xxp}} \bar{N}_{xx}^2 - \bar{N}_{xx} 2\pi R \, Le_{AV} \tag{27}$$

$$e_{AV} = a_1\bar{N}_{xx} + \frac{1}{L}\int_o^L \left\{ \frac{a_2}{d_{11}} \left[\frac{W_o}{R} + q_{11}W_o'' - \frac{n^2}{4R^2} \sum_{i=1}^{K} i^2 (W_i + 2W_i^o)W_i \right] \right.$$

$$\left. - a_3W_o'' + \frac{1}{2} W_o'(W_o' + W_o^{o}{}') + \frac{1}{4} \sum_{i=1}^{K} W_i'(W_i' + W_i^{o}{}') \right\} dx \tag{28a}$$

$$e = a_1\bar{N}_{xx} + \frac{1}{L}\int_o^L \Bigg\langle \frac{a_2}{d_{11}} \left[\frac{W_o}{R} + q_{11} W_o'' - \frac{n^2}{4R^2} \sum_{i=1}^{K} i^2 (W_i + 2W_i^o)W_i \right]$$

$$+ \sum_{i=1}^{2K} \left[a_1\left(\frac{in}{R}\right)^2 f_i - a_2f_i'' \right] - a_3 \sum_{i=0}^{K} W_i'' + a_4 \sum_{i=1}^{K} \left(\frac{in}{R}\right)^2 W_i$$

$$+ \frac{1}{2} \left[\sum_{i=0}^{K} W_i' \right] \sum_{i=0}^{K} (W_i' + 2W_i^{o}{}') \Bigg\rangle \; dx \tag{28b}$$

Finally, the appropriate boundary conditions are expressed in terms of W_i, W_i^o, and f_i. Note that, because of the character of the nonlinear differential field equations, Eqs. (6) and (7), by setting n = 0 one obtains the linearized version of equilibrium and compatibility. Furthermore, it is easily seen from Eqs. (24) that n = 1 includes the axisymmetric mode since the summation on i starts from zero. In addition, it is seen from the Fourier series representation of the imperfection that this

expression is suitable for the case when the imperfection is of the same shape as the buckling mode, as well as for any arbitrary symmetric (with respect to y) imperfection shape. In this latter case, in order to obtain a solution it is necessary to let n = 1 (in the series representation for W^o) and take K large enough for an accurate representation of the imperfection and for achieving a convergent solution. By employing Eq. (24) the expressions for the various boundary, symmetry and antisymmetry conditions become, Eqs. (23),

<u>SS-1</u> $W_o = W''_o = 0$

$$W_i = \gamma_1 W''_i + \gamma_4 f''_i = 0 \qquad i = 1,2,\ldots,K$$

$$f'_i = f_i = 0 \qquad i = 1,2,\ldots,2K$$

<u>SS-2</u> $W_o = W''_o - \gamma_3 \bar{N}_{xx} = 0$

$$W_i = \gamma_1 W''_i - \gamma_3\left[\bar{N}_{xx} + \left(\frac{in}{R}\right)^2 f_i\right] + \gamma_4 f''_i = 0 \qquad i = 1,2,\ldots,K$$

$$f'_i = 0 \qquad i = 1,2,\ldots,2K$$

$$b_2 f'''_i + b_3 W_i''' - \left(\frac{in}{R}\right)^2 b_4 W'_i + \frac{1}{R} W'_i$$

$$- \frac{n^2}{2R^2} \sum_{j=0}^{K} \left[(i+j)^2 W^o_{i+j} + (1 - \eta^2_{j-i} + \eta_i)(i-j)^2 W^o_{|i-j|}\right] W'_j = 0 \qquad (29a)$$

$$i = 1,2,\ldots,2K$$

<u>SS-3</u> $W_o = W''_o = 0$

$$W_i = \gamma_1 W''_i + \gamma_4 f''_i = 0 \qquad i = 1,2,\ldots,K$$

$$f_i = b_2 f_i'' + b_3 W_i'' = 0 \qquad i = 1,2,\ldots,2K$$

<u>SS-4</u>

$$W_o = W_o'' = 0$$

$$W_i = \gamma_1 W_i'' - \gamma_3\left[\left(\frac{in}{R}\right)^2 f_i + \bar{N}_{xx}\right] + \gamma_4 f_i'' = 0 \qquad i = 1,2,\ldots,K$$

$$-a_2\left[\bar{N}_{xx} + \left(\frac{in}{R}\right)^2 f_i\right] + b_2 f_i'' + b_3 W_i'' = 0 \quad ; \qquad i = 1,2,\ldots,2K$$

$$-\left[a_2 + 2/(1-\nu)E_{xxp}\right]\left(\frac{in}{R}\right)^2 f_i' + b_2 f_i'' + b_3 W_i''' - b_4\left(\frac{in}{R}\right)^2 W_i' + \frac{1}{R} W_i'$$

$$-\frac{n^2}{2R^2}\sum_{j=0}^{K}\left[(i+j)^2 W_{i+j}^o + (1 - \eta_{j-1}^2 + \eta_i) W_{|i-j|}^o\right] W_j' = 0$$

$$i = 1,2,\ldots,2K$$

<u>CC-1</u>

$$W_i = W_i' = 0 \quad , \qquad i = 0,1,2,\ldots,K$$

$$f_i' = f_i = 0 \quad , \qquad i = 1,2,\ldots,2K$$

<u>CC-2</u>

$$W_i = W_i' = 0 \quad , \qquad i = 0,1,2,\ldots,K$$

$$f_i' = b_2 f_i''' + b_3 W_i''' = 0 \quad , \qquad i = 1,2,\ldots,2K \tag{29b}$$

<u>CC-3</u>

$$W_i = W_i' = 0 \quad , \qquad i = 0,1,\ldots,K$$

$$f_i = b_2 f_i'' + b_3 W_i'' = 0 \quad , \qquad i = 1,2,\ldots,2K$$

<u>CC-4</u>

$$W_i = W_i' = 0 \quad , \qquad i = 0,1,2,\ldots,K$$

$$- a_2\left[\bar{N}_{xx} + \left(\frac{in}{R}\right)^2 f_i\right] + b_2 f_i'' + b_3 W_i'' = 0 \quad , \qquad i = 1,2,\ldots,2K$$

$$- \left[a_2 + 2/(1-v)E_{xxp}\right]\left(\frac{in}{R}\right)^2 f_i' + b_2 f_i''' + b_3 W_i''' = 0 \quad ,$$

$$i = 1,2,\ldots,2K$$

<u>FF-1</u>

$$W_o''\left[\gamma_1 - \gamma_4 q_{11}/d_{11}\right] - W_o\gamma_4/Rd_{11} + \frac{\gamma_4 n^2}{4R^2 d_{11}} \sum_{j=1}^{K} j^2(W_j + 2W_j^o)\, W_j = 0$$

$$W_o'''\left[\gamma_1 - \gamma_4 q_{11}/d_{11}\right] + W_o'\left[\bar{N}_{xx} - \gamma_4/Rd_{11}\right] + \bar{N}_{xx} W_o^{o\prime}$$

$$+ \frac{\gamma_4 n^2}{4R^2 d_{11}} \sum_{j=1}^{K} j^2\left[(W_j + 2W_j^o)\right]W_j' + (W_j' + 2W_j^{o\prime})W_j\Big] = 0$$

$$\gamma_1 W_i'' - \left(\frac{in}{R}\right)^2 \gamma_2 W_i + \gamma_4 f_i'' = 0 \quad , \qquad i = 1,2,\ldots,K \qquad (29c)$$

$$\gamma_4 f_i''' + \gamma_1 W_i''' - \left[\gamma_2 + 2D(1-\nu)\right]\left(\frac{in}{R}\right)^2 W_j' + \bar{N}_{xx}(W_i' + W_i^{o\prime}) = 0$$

$$i = 1,2,\ldots,K$$

$$f_i' = f_i = 0 \quad , \qquad i = 1,2,\ldots,2K$$

<u>Symmetry</u> $W_o' = 0$

$$W_o'''(\gamma_1 - \gamma_4 q_{11}/d_{11}) + \frac{\gamma n^2}{4R^2 d_{11}} \sum_{j=1}^{K} j^2\left[(W_i + 2W_j^o)W_j'\right.$$

$$+ (W_j' + 2W_j^{o'})W_j\Big] + \frac{n^2}{2R^2} \sum_{j=1}^{2K} \Big[\delta_{i+j} W_{i+j}^{o'} + (1-\eta_{j-1}^2 + \eta_i) W_{|i-j|}^{o'} \delta_{|i-j|}\Big] j^2 f_j = 0$$

$$W_i' = \gamma_1 W_i''' + \gamma_4 f_i''' + \frac{n^2}{2R^2} \sum_{j=1}^{2K} \Big[\delta_{i+j} W_{i+j}^{o'} + (1 - \eta_{j-i}^2 + \eta_i)\, \delta_{|i-j|} W_{|i-j|}^{o'}\Big] j f_i = 0$$

$$i = 1,2,\ldots,K \qquad (29d)$$

$$f_i' = b_2 f_i''' + b_3 W_i''' - \frac{n^2}{2R^2} \sum_{j=1}^{K} \Big[W_{i+j}^{o'} + (1 - \eta_{j-i}^2 + \eta_i) W_{|i-j|}^{o'}\Big] j^2 W_j = 0$$

$$i = 1,2,\ldots,2K$$

Antisymmetry: same as SS-3 (29e)

The solution procedure employed is described below.

A generalization of Newton's method (Refs. 20 and 21), applicable to differential equations, is employed to reduce the nonlinear field equations, Eqs. (25) and (26), and appropriate boundary conditions to a sequence of linear systems. In this method, the iteration equations are derived by assuming that the solution is achieved by a small correction to an approximate solution (initially taken as the linear solution). These small corrections are obtained through the solution of the linearized (with respect to the corrections) differential equations.

The linearized differential equations are written in matrix form as follows:

Field equations

$$[R]\{Z''\} + [S]\{Z'\} + [T]\{Z\} = \{g\} \qquad (30)$$

Boundary Conditions

$$[\bar{S}]\ \{Z'\} + [\bar{T}]\ \{Z\} = \{\bar{g}\} \tag{31}$$

where $\{Z\}$ is the vector of the 6K + 2 unknowns.

$$\{Z\}^T = \{W_o, W_1, \ldots W_K, f_1, f_2, \ldots f_{2K},\ W_o'', W_1'' \ldots, W_K'',\ f_1'', f_2'', \ldots f_{2K}''\} \tag{32}$$

Note that f_o has been eliminated in a manner similar to that of Refs. 13 and 14.

These ordinary differential equations are cast into the form of finite difference equations, and the system of ordinary equations, Eqs. (30) and (31), are changed into a system of linear algebraic equations. The usual central difference formula is used at all mesh points, i.e.

$$\begin{aligned} Z_\ell' &= (Z_{\ell+1} - Z_{\ell-1})/2\Delta \\ Z_\ell'' &= (Z_{\ell-1} - 2Z_\ell + Z_{\ell+1})/\Delta^2 \end{aligned} \tag{33}$$

Note that the second derivatives in W_i and f_i are taken as independent elements of the vector of the unknowns, therefore the second of Eqs. (33) applied only to fourth derivatives of W_i qnd f_i. By using one fictitious point on each side of the cylinder ends one obtains a system of (6K + 2) X (NP + 2) difference equations (NP - number of mesh points).

These equations are:

$$\begin{aligned} &[\bar{C}_1]\ \{Z_o\} + [\bar{B}_1]\ \{Z_1\} + [\bar{A}_1]\ \{Z_2\} = \bar{g}_1 \\ &[C_\ell]\ \{Z_{\ell-1}\} + [B_\ell]\ \{Z_\ell\} + [A_\ell]\ \{Z_{\ell+1}\} = g_\ell;\ \ell = 1,2,\ldots,NP \\ &[\bar{C}_{NP}]\ \{Z_{NP-1}\} + [\bar{B}_{NP}]\ \{Z_{NP}\} + [\bar{A}_{NP}]\ \{Z_{NP+1}\} = \bar{g}_{NP} \end{aligned} \tag{34}$$

where $\{Z_o\}$ and $\{Z_{NP+1}\}$ are unknown vectors at the two fictitious points, and the matrices in Eqs. (34) are given by

$$[\bar{C}_1] = -\frac{1}{2\Delta}[\bar{S}_1] \quad ; \quad [\bar{C}_{NP}] = -\frac{1}{2\Delta}[\bar{S}_{NP}]$$

$$[\bar{B}_1] = [\bar{T}_1] \quad ; \quad [\bar{B}_{NP}] = [\bar{T}_{NP}]$$

$$[\bar{A}_1] = \frac{1}{2\Delta}[\bar{S}_1] \quad ; \quad [\bar{A}_{NP}] = \frac{1}{2\Delta}[\bar{S}_{NP}]$$

$$\left.\begin{aligned} [C_\ell] &= \frac{1}{\Delta^2}[R_\ell] - \frac{1}{2\Delta}[S_\ell] \\ [B_\ell] &= -\frac{2}{\Delta^2}[\bar{R}_\ell] + [T_\ell] \\ [A_\ell] &= \frac{1}{\Delta^2}[R_\ell] + \frac{1}{2\Delta}[S_\ell] \end{aligned}\right\} \quad \ell = 1,2,\ldots,NP$$

The system of Eqs. (34) can be written, for all the shell mesh points, as

$$\begin{bmatrix} \boxed{\bar{C}_1}\boxed{\bar{B}_1}\boxed{\bar{A}_1} & & & & & \\ \boxed{C_1}\boxed{B_1}\boxed{A_1} & & & & 0 & \\ & \boxed{C_2}\boxed{B_2}\boxed{A_2} & & & & \\ & & \ddots & & & \\ & & \boxed{C_\ell}\boxed{B_\ell}\boxed{A_\ell} & & & \\ & & & \ddots & & \\ & 0 & & & \boxed{C_{NP-1}}\boxed{B_{NP-1}}\boxed{A_{NP-1}} & \\ & & & & & \boxed{C_{NP}}\boxed{B_{NP}}\boxed{A_{NP}} \\ & & & & & \boxed{\bar{C}_{NP}}\boxed{\bar{B}_{NP}}\boxed{\bar{A}_{NP}} \end{bmatrix} \begin{Bmatrix} Z_o \\ Z_1 \\ Z_2 \\ \vdots \\ Z_\ell \\ \vdots \\ Z_{NP-1} \\ Z_{NP} \\ Z_{NP+1} \end{Bmatrix} = \begin{Bmatrix} \bar{g}_1 \\ g_1 \\ g_2 \\ \vdots \\ g_\ell \\ \vdots \\ g_{NP-1} \\ g_{NP} \\ \bar{g}_{NP} \end{Bmatrix} \quad (35)$$

This system is solved by the special algorithm which is reported in Ref. 22.

When the load parameter is at a limit point a unique solution does not exist (the system becomes singular) and thus the solution does not converge. Therefore, the solution procedure goes as follows: first, the system of equations is solved for a small level of the applied load (say 20% of the classical buckling load), then a multiple of this solution is used for a small increase in the load parameter until the process fails to converge. The load level at which the solution fails to converge is taken to be the critical load. Note that, when approaching the limit load, if the increment in the load value is large enough so as to place the systems at equilibrium far beyond the limit point, the system in some cases, does converge. In this case, since the interest is in the limit point value only, one can check the sign of the determinant of the coefficients of the unknown vector. If the sign changes, by taking a large increment in the load level, one must decrease the increment and proceed with the solution. Large increments are used in the procedure in order to save computer time. Because of the use of large increments and since, in some cases, the solution converged at both consecutive steps the criterion of the change in the determinant sign is employed to establish the existence of a critical point within the range of these consecutive steps. At each level of the load for which the sytem is solved, the value of the number of full circumferential waves is needed. Different values of n are used to obtain a solution, and the one that minimizes the total potential, Eq. (27), is taken as the correct one (see Refs. 13 and 14). Numerical integration is used to find the total potential.

The number of n values to be tried at every increment of the load is small, since the circumferential mode does not vary significantly with small increases in the load, $\bar{N}_{xx}$. The end shortenings at each level of the applied load are also computed through numerical integration.

Chapter IV

APPLICATIONS AND DISCUSSION

The mathematical formulation and the method of solution for the buckling analysis of imperfect, thin, circular, stiffened cylindrical shells under uniform axial compression is presented in Chapters II and III. The methodology is demonstrated through a number of illustrative examples. Numerical solutions are obtained by employing the Georgia Tech high speed digital computer CDC-CYBER 70, Model 74-28. A general program is written (see Appendix B) which includes the following desirable features: (a) it is applicable to stiffened (in either or both directions) geometries as well as unstiffened; (b) it accomodates all possible boundary conditions (SS,CC,FF, etc.), and it can easily be modified to accomodate elastic end restraints; (c) the number of Fourier terms (K) can be as large as desired. The same holds true for the number of points (NP) in the finite difference scheme; (d) the geometric imperfection can be axisymmetric as well as an arbitrary symmetric (w.r.t. y) one. The program can easily be modified to include other destabilizing loading conditions such as pressure and torsion.

Although the program is highly dimensional because of the number of Fourier terms, number of points and number of required iterations, the solution is obtained with reasonable CPU time. For example, by using $K = 1$ (one-term) and 65 points (536 unknowns) it requires four seconds to complete one iteration; for the same but $K = 2$ it requires 15 seconds. For a convergent solution to be obtained at low levels of the applied load two iterations are sufficient. At load levels approaching the limit

point six iterations are needed (convergence: percent difference $< 10^{-4}$).

The numerical results for all the illustrative examples are presented in tabular form in Table 1. A number of these examples are taken from the open literature in order to check the present solution. In addition, new results are generated and the discussion of both is given below. In this table, for each example considered, the buckling load (see columns of $\bar{N}_{xx_{cr}}$ and $\bar{N}_{xx_{cr}}/N_{xx_{c\ell}}$) is bracketed between two numbers (denoting the desired accuracy). The first number denotes the highest level of the load for which a convergent solution is obtained and the second number a level higher than the limit point (according to criterion discussed herein). In all examples, the imperfection is taken to be symmetric with respect to x = L/2 and therefore the response is taken to be symmetric. Thus only half of the cylinder is analyzed by employing the appropriate symmetric conditions at x = L/2. The examples can broadly be classified in one of the following four categories: (a) unstiffened (Examples 1-5); (b) stringer-stiffened (Examples 6-9); (c) ring-stiffened (Examples 10 and 11); and (d) ring- and stringer-stiffened (Examples 12-21). In each of the four categories, at least one case was calculated for two different truncated Fourier series (K = 1 and K = 2) and for two different number of points (NP = 35 and NP = 65) in order to check the effect of these two parameters on the convergence of the solution.

(a) Unstiffened (Examples 1-5). This geometry is taken from Ref. 16. Only Example 3 is reported in Ref. 16 and the results are in very good agreement. Examples 1, 2, 4 and 5 are considered in order to assess the effect of boundary conditions for this type of an imperfection (see Table

1). It is seen from the results that the effect of in-plane boundary conditions is significant for the simply supported case and insignificant for the clamped case. In Example 2, because of the u = C in-plane boundary condition, the criterion used for finding $\bar{N}_{xx_{cr}}$ is that the determinant (see Chapter 3) goes to zero. It can be seen from Fig. 2 that as the sign of the determinant changes the radial mode of deformation changes. The calculations for this case were based on K = 1 and 65 grid points in the axial direction (for half the cylinder length).

(b) Stringer-stiffened (Examples 6-9). The geometry for these examples is taken from Ref. 9 and referred to, in it, as heavy stringers. The present results are in very good agreement with those of Ref. 9. Examples 6 and 7 correspond to Z = 95.4 with external stringers and the imperfection shape is virtually axisymmetric in Example 6, and symmetric in Example 7. The axisymmetric imperfection yields greater reduction than the symmetric one. Note that, for this case, the perfect geometry buckles axisymmetrically. Examples 8 and 9 correspond to Z = 394 with external and internal stringers respectively and an imperfection which is primarily axisymmetric. From these examples one can conclude (as in Ref. 9) that imperfection sensitivity is greatly affected by the curvature parameter Z, and that externally stiffened configurations are much more sensitive to geometric imperfections than internally stiffened ones. These cases were analyzed with 35 and 65 grid points in the axial direction for half the cylinder length. From these computations one can see (for a typical case, see Tables 2 and 3), that the critical load and the response (w,F) for each n are almost the same regardless of the number

of points.

(c) Ring-stiffened (Examples 10 and 11). This geometry is also taken from Ref. 9, and it corresponds to light ring-stiffened cylinders with Z = 394 and external and internal positioning of the rings respectively. Although the numerical results of the present analysis are in good agreement with those of Ref. 9, the conclusion concerning the effect of ring positioning on the sensitivity is reversed. According to the present results internal rings make the overall configuration more sensitive than external rings.

(d) Ring and stringer stiffened (Examples 12-21). These examples are chosen to demonstrate the methodology for ring and stringer stiffened configuration. In addition, the geometries were chosen in such a way that some comparison can be given with stiffened configurations of the only stringer or ring stiffened type. These examples correspond to a geometry for which Z = 95.4, $(e_x/t) = \pm 6$, $(e_y/t) = \pm 3$, $\bar{\lambda}_{xx} = 0.455$, $\bar{\rho}_x = 100$, and $\bar{\rho}_y = 20$ (see Table 1). The imperfection shape is taken to be similar to the buckling mode and the imperfection amplitude is varied from half to four times the thickness of the skin. The reader is reminded that the analysis is based on the smeared technique. According to Ref. 9 this configuration without rings is highly sensitive to geometric imperfections when the stringers are positioned on the outside and virtually insensitive for inside positioning of the stringers. The present results (see Table 1) indicate that with the presence of rings the sensitivity is reduced for external stiffeners and aggrevated (increased) for internal stiffeners. Examples 12-16 correspond to external stiffening, while

examples 17-21 to internal stiffening. The configuration is checked for a primarily axisymmetric imperfection (examples 12 and 17) as well as symmetric imperfections (remaining examples). The effect of the imperfection amplitude is checked for symmetric imperfections and both external and internal stiffening. This effect is shown graphically on Fig. 3. Fig. 4 shows the plots of load versus "unit and shortening" for three amplitudes of the symmetric imperfection and external stiffeners. Table 2, which corresponds to example 14 but is typical for all examples considered, depicts in tabular form the computational procedure required to arrive at the final results shown in Table 1. From this table one can see that the minimum total potential corresponds to n = 4. For this n value, all results are the same for NP = 35 and NP =65. In addition, when K =1 and K = 2, the critical load differs by 4% or less. Table 3 presents a comparison of response (radial displacement and stress function amplitudes) and critical loads for different K values (number of Fourier terms) and two different values of NP (number of grid points).

Table 1: Final Results for Imperfect Unstiffened and Stiffened Cylindrical Shell.

Example No.	$E = 10.5\cdot 10^6$ $\nu = 0.3$ $R = 4.0$ Geometric parameters								Boundary Condition	linear buckling load perfect cylinder			Imperfection $w^o(x,y)$	Nonlinear limit point imperfect cylinder		
	L	t	$\frac{e_x}{t}$	$\frac{e_y}{t}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	$\bar{\rho}_{xx}$	$\bar{\rho}_{yy}$		$N_{x_{cl}}$	n	m		$N_{x_{cr}}$	$N_{x_{cr}}/N_{x_{cl}}$	n
1	4.	0.004	0	0	0	0	0	0	SS1	14.48*			$-0.5t \cos\frac{2\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	12.50 ÷ 14.50	0.860 ÷ 1.000	12
2	4.	0.004	0	0	0	0	0	0	SS2	14.48*			$-0.5t \cos\frac{2\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	14.00 ÷ 16.00	0.967 ÷ 1.105	15
3	4.	0.004	0	0	0	0	0	0	SS3	25.42			$-0.5t \cos\frac{2\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	16.50 ÷ 16.56	0.649 ÷ 0.651	13
4	4.	0.004	0	0	0	0	0	0	CC1	25.42			$-0.5t \cos\frac{2\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	16.88 ÷ 17.00	0.664 ÷ 0.669	13
5	4.	0.004	0	0	0	0	0	0	CC3	25.42			$-0.5t \cos\frac{2\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	16.88 ÷ 17.00	0.664 ÷ 0.669	13
6	4.	0.04	-15.	0	1.82	0	1000.	0	SS3	123300.	0	1	$2t \sin\frac{\pi x}{L} + 0.2t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	73000. ÷ 78000.	0.592 ÷ 0.632	7
7	4.	0.04	-15.	0	1.82	0	1000.	0	SS3	123300.	0	1	$2t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	101450. ÷ 111450.	0.823 ÷ 0.903	1
8	8.125	0.04	-15.	0	1.82	0	1000.	0	SS3	58480.	5	1	$2t \sin\frac{\pi x}{L} + 0.2t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	19000. ÷ 20000.	0.325 ÷ 0.341	12
9	8.125	0.04	-15.	0	1.82	0	1000.	0	SS3	24810.	3	1	$2t \sin\frac{\pi x}{L} + 0.2t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	24000. ÷ 25000.	0.967 ÷ 1.007	3
10	8.125	0.04	0.	-3.	0.	0.455	0.	20	SS3	3140.	2	13	$0.6t \sin\frac{m\pi x}{L} + 0.06t \sin\frac{m\pi x}{L}\cos\frac{ny}{R}$	1700. ÷ 1800.	0.541 ÷ 0.573	2
11	8.125	0.04	0.	3.	0.	0.455	0.	20	SS3	2713.	5	11	$0.6t \sin\frac{m\pi x}{L} + 0.06t \sin\frac{m\pi x}{L}\cos\frac{ny}{R}$	1600. ÷ 1725.	0.589 ÷ 0.635	2
12	4.	0.04	-6.	-3.	0.91	0.455	100.	20	SS3	35220.	4	1	$0.5t \sin\frac{\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	26500. ÷ 28000.	0.752 ÷ 0.795	4
13	4.	0.04	-6.	-3.	0.91	0.455	100.	20	SS3	35220.	4	1	$0.5t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	25000. ÷ 26500.	0.710 ÷ 0.752	4
14	4.	0.04	-6.	-3.	0.91	0.455	100.	20	SS3	35220.	4	1	$1.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	20500. ÷ 22000.	0.582 ÷ 0.625	4
15	4.	0.04	-6.	-3.	0.91	0.455	100.	20	SS3	35220.	4	1	$2.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	16000. ÷ 16750.	0.454 ÷ 0.476	4
16	4.	0.04	-6.	-3.	0.91	0.455	100.	20	SS3	35220.	4	1	$4.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	13000. ÷ 13500.	0.369 ÷ 0.383	4
17	4.	0.04	6.	3.	0.91	0.455	100.	20	SS3	19790.	4	1	$0.5t \sin\frac{\pi x}{L} + 0.05t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	17000. ÷ 17500.	0.859 ÷ 0.884	4
18	4.	0.04	6.	3.	0.91	0.455	100.	20	SS3	19790.	4	1	$0.5t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	17000. ÷ 17500.	0.859 ÷ 0.884	4
19	4.	0.04	6.	3.	0.91	0.455	100.	20	SS3	19790.	4	1	$1.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	15250. ÷ 15630.	0.771 ÷ 0.790	4
20	4.	0.04	6.	3.	0.91	0.455	100.	20	SS3	19790.	4	1	$2.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	13750. ÷ 14500.	0.695 ÷ 0.733	4
21	4	0.04	6.	3.	0.91	0.455	100.	20	SS3	19790.	4	1	$4.0t \sin\frac{\pi x}{L}\cos\frac{ny}{R}$	11200. ÷ 11400.	0.566 ÷ 0.576	3

*Estimated as 0.57 $N_{xx_{cl}}$ (for SS3) [Ref. 14].

Table 2. Computations for Estimating n and $\bar{N}_{xx_{cr}}$ (Example 14).

$\bar{N}_{xx}$	$\frac{\bar{N}_{xx}}{N_{x_{c\ell}}}$	n	k	Points NP	Total Potential	"end shortening"	No. of Iterations	CPU TIME (SEC)
19000	0.54	3	1	35	-10777.	0.021870	3	9
22000	0.62	3	1	35	-14499.	0.025424	4	11
23500	0.67	3	1	35	-16590.	0.027394	4	12
25000	0.71	3	1	35	over limit point			
13000	0.37	4	1	35	- 5028.	0.015124	3	9
16000	0.45	4	1	35	- 7628.	0.018620	3	9
19000	0.54	4	1	35	-10785.	0.022161	4	11
22000	0.62	4	1	35	-14534.	0.026189	6	20
22750	0.65	4	1	35	over limit point			
19000	0.54	4	1	65	-10784.	0.022160	4	19
22000	0.62	4	1	65	-14534.	0.026188	6	33
22750	0.65	4	1	65	over limit point			
19000	0.54	4	2	65	-10789.	0.022232	4	70
20500	0.58	4	2	65	-12590.	0.024197	5	90
22000	0.62	4	2	65	over limit point			
19000	0.54	5	1	35	-10745.	0.022248	4	11
22000	0.62	5	1	35	-14443.	0.025847	4	11
22750	0.65	5	1	35	-15459.	0.026813	5	15
23500	0.67	5	1	35	over limit point			
22000	0.62	6	1	35	-14362.	0.025764	4	11
22000	0.62	7	1	35	-14318.	0.025762	3	9
22000	0.62	12	1	35	-14267.	0.025758	2	7

Table 3. Effect of Number of Fourier Terms and Grid Points on the Response Amplitudes.

$$w^{o}(x,y) = \delta_2 \sin \frac{\pi x}{L} \cos \frac{ny}{R}$$

δ_2/t	K	No. of Points	Examples	$\frac{N_{x_{cr}}}{N_{x_{c\ell}}}$	w_0	w_1	w_2	f_1	f_2	f_3	f_4
1.0	1	35	14	0.539	0.009172	0.052284	--	2246.40	-77.13	--	--
1.0	1	65		0.539	0.009167	0.052270	--	2245.88	-77.12	--	--
1.0	2	65		0.539	0.009894	0.055149	0.0035991	2306.74	-39.86	-4.55	-0.45
1.0	1	35		0.625	0.024436	0.096853	--	3976.12	-191.04	--	--
1.0	1	65		0.625	0.024401	0.096768	--	3973.14	-190.83	--	--
1.0	2	65		0.582	0.015810	0.074324	0.0058296	3018.11	-60.28	-8.86	-0.12
1.0	2	65		0.625	over limit point		→ 3% Difference				
1.0	1	65		0.646	over limit point						
2.0	1	35	15	0.454	0.026022	0.086314	--	3405.72	-236.92	--	--
2.0	1	65		0.454	0.026005	0.086281	--	3404.66	-236.86	--	--
2.0	2	65		0.454	0.034229	0.105602	0.0114593	3699.64	-194.83	-28.22	-0.4501
2.0	1	35		0.497	over limit point						
2.0	1	65		0.497	over limit point		→ 4% Difference				
2.0	2	65		0.476	over limit point						
4.0	1	35	16	0.369	0.113547	0.201740	--	4794.98	-1166.29	--	--
4.0	2	65		0.369	0.132664	0.230965	0.21715	3549.32	-1342.51	-109.53	-1.37
4.0	1	35		0.383	over limit point						
4.0	2	65		0.383	over limit point						
1.0	1	35	19	0.771	0.036821	0.164462	--	1483.81	-447.37	--	--
1.0	2	65		0.771	0.036716	0.164500	0.002221	1409.59	-461.42	-5.957	-0.0159
1.0	1	35		0.790	over limit point						
1.0	2	65		0.790	over limit point						

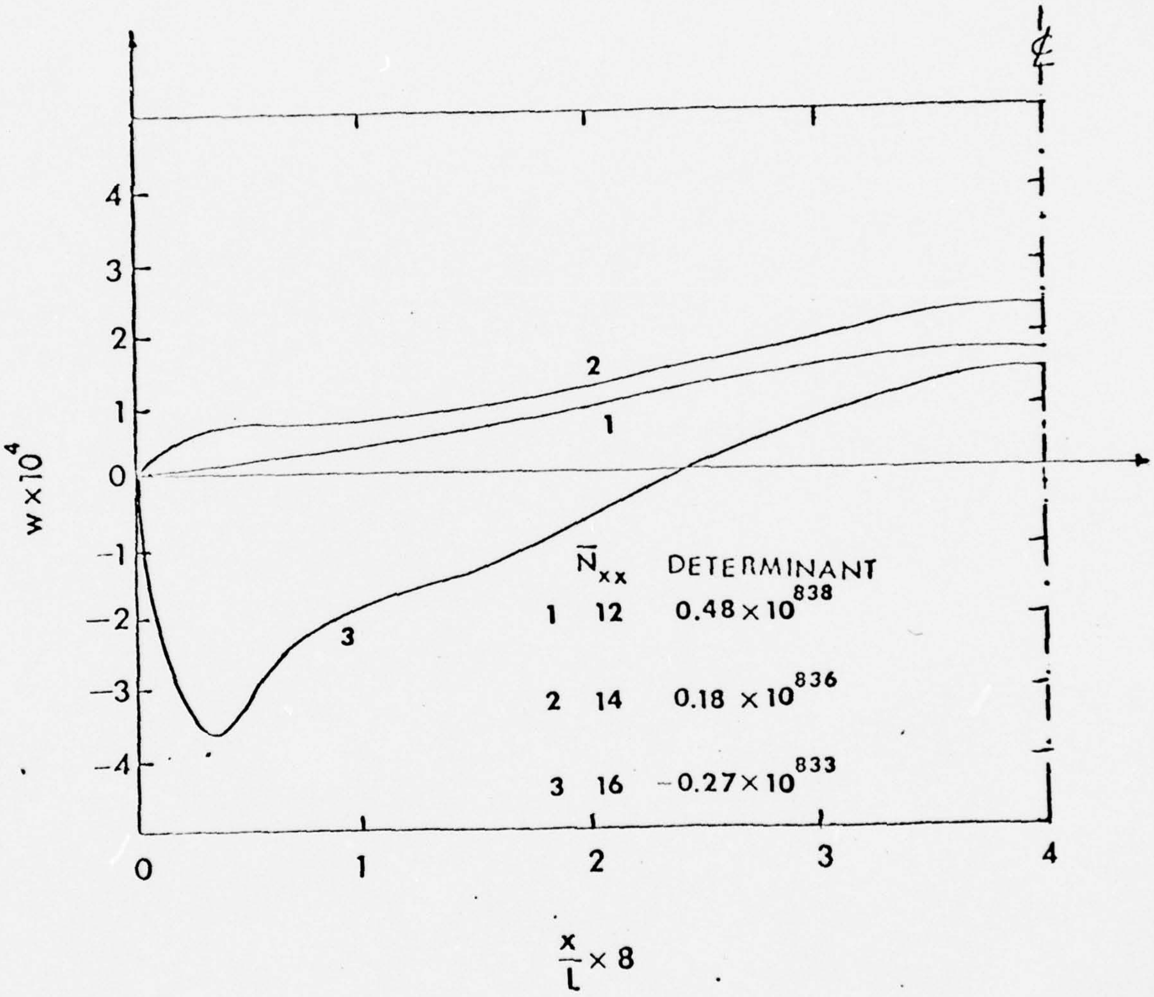


Fig. 2. Radial Dispalcement along y = 0 (Example 2).

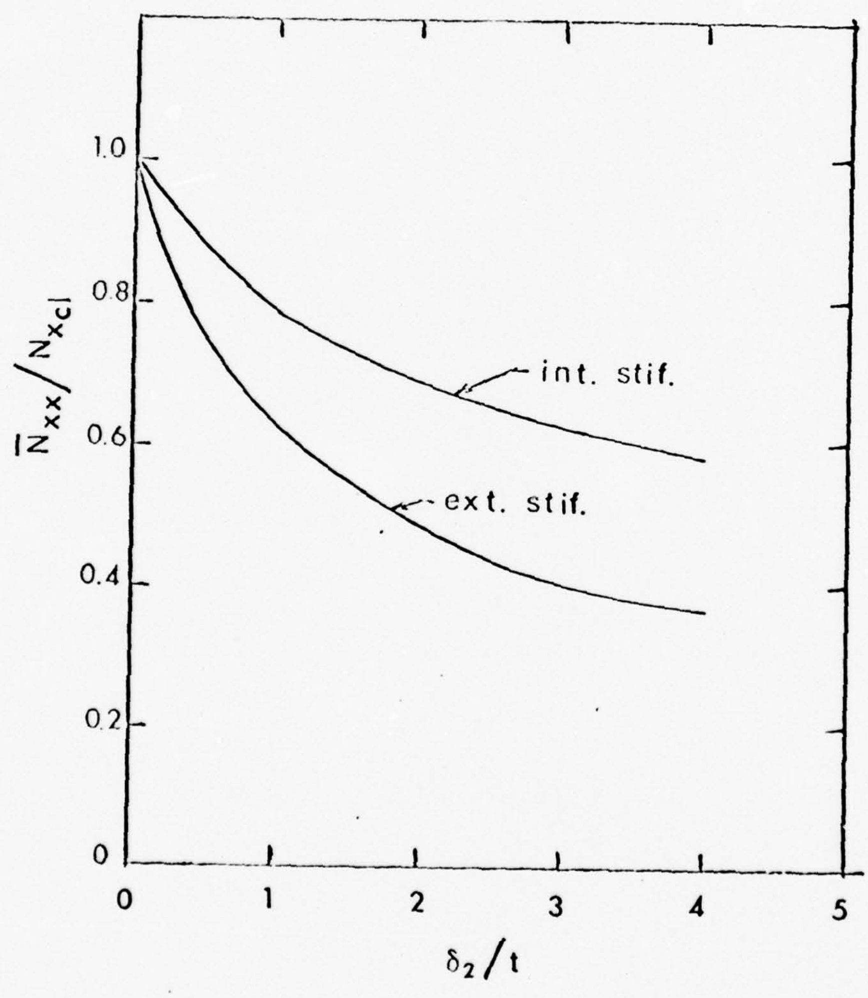


Fig. 3. Effect of Amplitude of Symmetric Imperfection on the Critical Load (Examples 13-16 and 18-21).

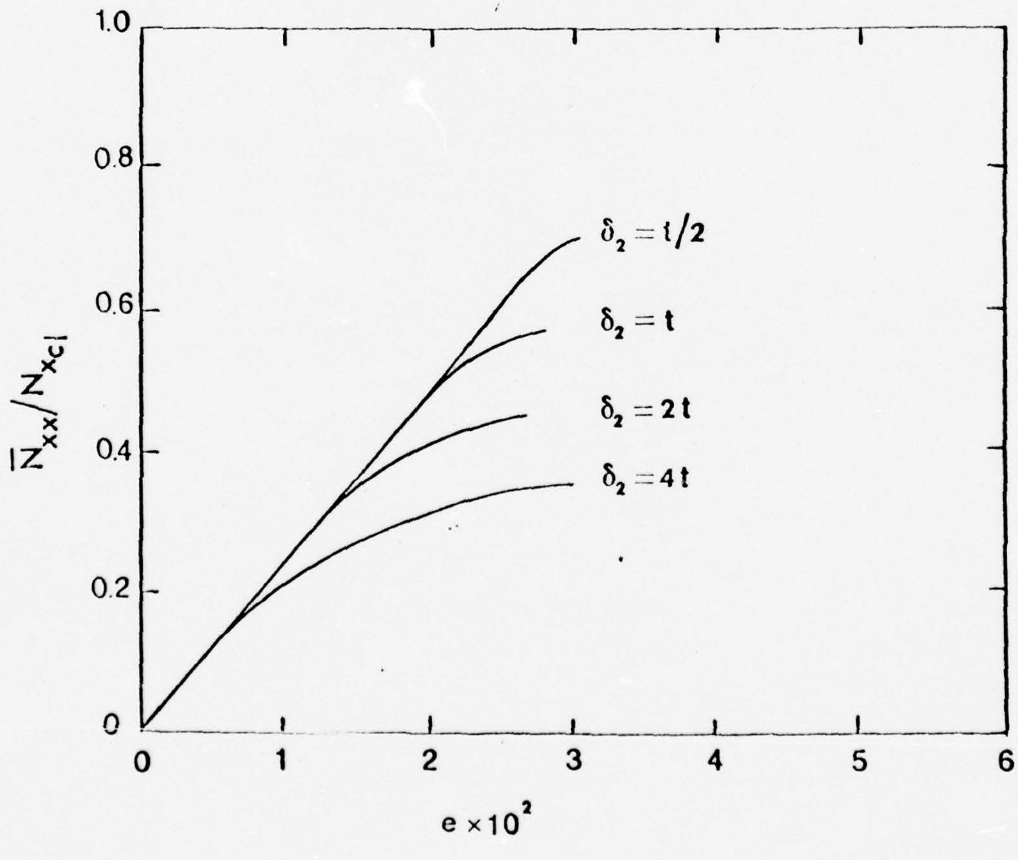


Fig. 4. Load versus "unit end shortening" (Examples 13-15).

Chapter V

CONCLUSIONS BASED ON THE NONLINEAR BUCKLING ANALYSIS

A methodology for the buckling analysis of imperfect, thin, circular, cylindrical, stiffened shells under uniform axial compression and for various end conditions is presented and demonstrated through a number of examples. On the basis of the results reported herein very few, if any, general conclusions can be drawn. Among these one may list the following:

(1) It seems that the imperfection sensitivity of generally stiffened configurations strongly depends on the curvature parameter.

(2) From the examples considered, it appears that configurations with external stiffening are more imperfection sensitive than configurations with internal stiffening.

(3) The few examples considered seem to support the contention that the most severe shape of imperfection is that which resembles the buckling mode.

(4) The curve that demonstrates the effect of the imperfection amplitude on the critical load seems to be approaching a finite asymptote (see Fig. 3).

(5) The presence of both stringers and rings in a configuration alters the conclusions regarding the effects of positioning of the stiffeners and of the curvature parameter on the imperfection sensitivity of a configuration with either strings or rings only. The limited generated data suggests a nonlinear coupling of the individual effects.

(6) In general one should not expect the wave number, n, corresponding to the limit point for an imperfect cylinder, to be the same as the one

predicted by the linear analysis of the corresponding perfect geometry. This difference, though, seems to be minimized when the configuration is stiffened in both directions (see Table 1; Examples 12-21).

Chapter VI

IMPERFECTION SENSITIVITY OF OPTIMAL CYLINDERS

The formulation of minimum weight design or optimization of stiffened cylinders under destabilizing loads is based on classical (linear) buckling analyses (see References 18 and 23 and references therein). A knockdown factor is used to account for the fact that such geometries are sensitive to geometric imperfections. Such formulation and design procedure is fully described and demonstrated through a number of examples in References 18 and 23.

The precise statement of the problem considered in the above two references (in its most general form) is as follows: Given an internally stiffened thin circular, cylindrical shell of specified material, radius, and length, find the size, shape, and spacings of the stiffeners, and the thickness of the skin such that the resulting configuration can safely carry a given axial compressive load with minimum weight. Since this configuration is sensitive to geometric imperfections, a true solution can be accomplished by incorporating, in the design procedure, a nonlinear buckling analysis (of geometrically imperfect cylinders - as described in Chapters II and III). This task is formidable and it will probably require an unreasonably large amount of computer time.

The reasonable alternate to the above approach is to follow the design procedure employed in References 18 and 23, establish the optimum point in the design space and then perform an imperfection sensitivity analysis (nonlinear buckling analysis) on the optimum point as well as the surrounding design space in order to establish what effect the

different design variables have on the knockdown factor. The design space that needs to be investigated, for a given amplitude of the imperfection (the shape can either be considered the same for all design points, or it can be taken to be as one that leads to the largest reduction in load - see Chapter IV), should be established apriori by employing the following criterion: if for a given problem the optimum weight is 500 lbs., as one moves away from this optimum geometry (variations in the design variables) the weight increases; in addition, it is well accepted that stiffened cylinders are not as sensitive as unstiffened ones (on the basis of available experimental data); therefore, if for a given imperfection the critical load is 60% of the corresponding perfect geometry critical load, (knockdown factor = .6), then the design space in which comparison studies are made should be such that its boundary weight is no larger than, say 50%, of the optimum weight; this way one can find out if whatever is gained by being at an optimum point (on the basis of linear buckling analysis) is not lost because of large variations in the knockdown factor as one moves away from the optimum design point. The only question in this procedure is whether one needs to use one value of the load and one knockdown factor in the linear optimization procedure or several in order to establish how the optimum design point moves in the design space with small variations (10-15%) in the $\bar{N}_{xx}$ value used in References 18 and 23. This question is dealth with in the section entitled "Recommended Design Procedure."

1. Design Examples

This alternate approach is employed herein and it is applied to the following two design cases of References 18 and 23.

Case 1. (Ref. 18) $R = 95.5$ in., $L = 291$ in., $\bar{N}_{xx} = 800$ lb/in., $E = 10.5 \times 10^6$ psi; $\rho = 0.101$ lbs/in.^{3}, $\nu = 0.33$,

$\delta = 0.0442$ in. (imperfection amplitude)

RSRR (rectangular stringers and rings)

MG = 0.02 in. (minimum gage constraint)

Boundary Conditions SS3.

Case 2. (Ref. 23) $R = 85$ in., $L = 100$ in., $\bar{N}_{xx} = 2700$ lbs/in., $E = 10.5$ psi, $\rho = 0.101$ lbs/in.^{3}, $\nu = 0.33$

$\delta = 0.1$ in., RSRR (rectangular stringer and rings),

TSRR (T stringers and rectangular rings with

$C_x = 1.079$ and 1.135)

MG = 0.05 in., Boundary Conditions SS3.

In both cases the imperfection amplitude is almost twice the skin thickness of the optimum geometry. Both cases are checked for an axisymmetric and for a symmetric shape of the imperfection. The results for the latter shape are reported herein, because they yield the greatest sensitivity. This shape is given by

$$w^o(x,y) = \delta \sin \frac{m\pi x}{L} \cos \frac{ny}{R} .$$

Case 1 corresponds to a geometry with rectangular stringers and rings (RSRR) and the optimum design geometry is reported in References 18. Case 2 corresponds to a geometry with tee stringers and rectangular rings

(optimum stiffener shapes) with various flange width to web height ratios ($k_x = 0.65$, $C_x = 1.212$; $k_x = 0.45$, $C_x = 1.135$; $k_x = 0.30$, $C_x = 1.079$) including the limiting case of $k_x = 0$ and $C_x = 1.000$ which corresponds to rectangular stringers. The corresponding geometries are reported in Table 6 of Reference 23.

The results are presented in tabular form and are discussed below.

2. <u>Discussion of Results</u>

<u>Case 1.</u> The important results associated with the investigation of Case 1 are presented in Table 4. The minimum weight configuration obtained in Reference 18 is labeled as point 2 on this table. The knockdown factor for this point is found to be 0.9125 and the shape of the imperfection that yields the greatest reduction corresponds to the perfect geometry buckling mode, i.e., m = 17 and n = 9. Because of this, the imperfection for all design points in the neighborhood of the optimum is taken to be $0.0442 \sin \frac{17\pi y}{L} \cos \frac{ny}{R}$, and n for each point is a free parameter and evaluated such that it yields the most reduction from the classical buckling load. Points 1, 3, and 4 correspond to the optimal geometries for spanning values of the curvature parameter Z. Note that as Z decreases the knockdown factor decreases, which means that the thicker the shell the bigger the imperfection sensitivity. By comparing the weights and the knockdown factors for these four points, it can easily be concluded that point 2 geometry is better than the geometry corresponding to points 3 and 4. The only question exists with point 1 geometry since it is less sensitive than that of point 2. But, since the weight of point

1 geometry is heavier than that of point 2 by 5.5% while the knockdown factor is only higher by 2.8%, one might still conclude that the optimal geometry (point 2) is the best even in the presence of imperfection sensitivity. At this point the authors would like to point out that these conclusions do not suffer by keeping m = 17 in the imperfection shape, because points 3 and 4, only, might be affected by letting m be a free parameter, in which case the knockdown factor for these points might become smaller.

Once the effect of the curvature parameter is established, the investigators considered the effect of the other design parameters, such as extensional and bending stiffnesses $(\alpha_x, \alpha_y, \lambda_{xx}, \lambda_{yy})$. For the Z value that corresponds to the optimal geometry (point 2) the design space surrounding the optimal point is investigated (in the presence of imperfections) and the comparison of the weights and knockdown factors (points 2a, 2b, 2c, 2d, 2e and 2) supports the conclusion that point 2 yields the minimum weight design even when imperfections are present. In this comparison it is also observed that small variations in all other design parameters, except the curvature parameter Z, have a very small effect on the knockdown factor. A similar investigation of the surrounding design space of point 1 (Z = 41,850) yields the same conclusion (these results are not reported herein).

Case 2. The results of the investigation for this case are reported in Table 5. The imperfections are takens as A) $w^o = 0.1 \sin \frac{5\pi x}{L} \cos \frac{ny}{R}$ and B) $w^o = 0.1 \sin \frac{m\pi x}{L} \cos \frac{ny}{R}$. The optimal geometry corresponds to tee stringers and rectangular rings and is labeled as point 1 in Table 5 (see

Table 6 of Ref. 23). Two shapes for the imperfection are considered , one denoted by 0.1 sin $\frac{5\pi x}{L}$ cos $\frac{ny}{R}$ and the other by 0.1 sin $\frac{m\pi x}{L}$ cos $\frac{ny}{R}$, where m in the second is taken to be the classical buckling mode number of half sine waves in the axial direction and n (in both cases) corresponds to the number of full waves in the circumferential direction that yields the smallest buckling load (in the presence of the imperfection). The imperfection amplitude in both cases is twice the thickness of the optimum geometry. Point 2 corresponds to the optimum geometry at a lower Z value. For both points (1 and 2) the geometry corresponds to tee stringers and rectangular rings (TSRR). Point 3 employs rectangular stiffeners and corresponds to the optimum geometry for this construction and the same Z as point 1. Points 1a through 1g correspond to the same construction (TSRR) and Z value as point 1 but different values of the remaining design parameters $(\bar{\alpha}_x, \bar{\alpha}_y, \lambda_{xx}, \lambda_{yy}, C_x)$. Finally, points 3a through 3e correspond to the same construction (RSRR) and Z value as point 3 but different values of the remaining design parameters.

Optimal geometries corresponding to higher Z values than point ; are checked, but since the knockdown factor for these geometries is smaller than that of point 1, the results are not presented herein. This conclusion is the same as that of Case 1.

In making the comparison studies the smallest of the knockdown factors is considered.

In comparing points 1 and 2, the results are inconclusive because the difference in knockdown factors and that of the corresponding weights are approximately the same. This might be interpreted as both geometries

being very close to the optimum for this construction.

A comparison of the data for points 1a-1g with those of point 1 suggests that point 1f might yield a lighter configuration than that of point 1. The reason for this conclusion is based on the observation that the geometry of point 1f is heavier than that of point 1 by 1% while it is less sensitive by approximately 15%.

A similar comparison among the RSRR geometries shows that point 3 geometry is indeed the best geometry (same conclusion as in Case 1).

Finally, a comparison between the two different constructions suggests that the best of either corresponds to an acceptable design. The only thing in favor of the RSRR construction is that, if cost of manufacturing is taken under consideration (additional constraint), then this construction is superior.

Note that in both cases the results of the linear optimization procedure, corresponding to only one value of $\bar{N}_{xx}$, are employed. This may cast some doubt to the validity of the conclusion and for this reason an improved design methodology is suggested in the next chapter.

Table 4. Effect of Imperfections on the Optimal Geometry (CASE 1).

$$\left[w^{o}(x,y) = 0.0442 \sin \frac{17\pi x}{L} \cos \frac{ny}{R} \right]$$

	Point	1	2	3	4	2a	2b	2c	2d	2e
	W, lbs.	801	755	760	790	770	789	832	809	792
Geometric parameters	h, in.	0.02	0.0221052	0.024	0.028	0.0221052	0.0221052	0.0221052	0.0221052	0.0221052
	$\bar{\alpha}_x$	21	20	18	15	20	20	20	20	25
	$\bar{\alpha}_y$	120	95	85	65	90	85	75	80	55
	λ_{xx}	0.8690	0.6319	0.5276	0.3962	0.6384	0.6578	0.7227	0.6904	0.5711
	λ_{yy} *	0.2636	0.2049	0.1833	0.1391	0.2325	0.2546	0.2885	0.2686	0.3493
	ρ_{xx} *	383.23	252.79	170.95	89.15	255.37	263.14	299.09	276.16	356.93
	ρ_{yy} *	3796.40	1848.86	1324.27	587.82	1883.17	1839.20	1622.76	1718.85	1056.60
	e_x *	0.220	0.232	0.228	0.224	0.232	0.232	0.232	0.232	0.287
	e_y *	1.210	1.061	1.032	0.924	1.005	0.951	0.840	0.895	0.619
	Z	41850	37870	34880	29894	37870	37870	37870	37870	37870
perfect cylinder	$N_{x_{cl}}$	800	800	800	800	800	800	300	800	800
	m	17	17	18	19	17	17	17	17	14
	n	8	9	10	11	9	9	10	10	10
imperfect cylinder	$N_{x_{cr}}$	755	730	720	695	730	725	735	735	735
	n	8	9	10	11	9	9	10	10	10
	$N_{x_{cr}}/N_{x_{cl}}$	0.9438	0.9125	0.9000	0.8690	0.9125	0.9062	0.9187	0.9187	0.9187

*The ρ's and e's can be found from the α's and λ's (see Ref. 18).

Table 5 . Effect of Imperfections on the Optimal Geometry (CASE 2).

$$\left[A;\ w^{o} = 0.1 \sin \frac{5\pi x}{L} \cos \frac{ny}{L} \quad \cdot \quad B;\ w^{o} = 0.1 \sin \frac{m\pi x}{L} \cos \frac{ny}{R}\right]$$

	Point →	TS 1	TS 2	TS 3	TS 1a	TS 1b	TS 1c	TS 1d	TS 1e
	W, lb.	473	491	486	499	474	484	488	500
	h, in.	0.0500	0.0550	0.05	0.05	0.05	0.05	0.05	0.05
	$\bar{\alpha}_x$	16	11	12	15	15	17	13	12
	$\bar{\alpha}_y$	35	45	60	40	35	35	40	45
	λ_{xx}	0.5386	0.3310	0.4608	0.6629	0.4940	0.5990	0.4525	0.4508
	λ_{yy}	0.1351	0.2128	0.2309	0.0961	0.1804	0.1105	0.2685	0.3119
	ρ_{xx} *	137.88	40.04	66.35	149.14	111.14	173.11	76.47	64.91
	ρ_{yy} *	165.45	430.88	831.17	153.74	220.95	135.38	429.66	631.50
	e_x *	0.457	0.354	0.325	0.429	0.429	0.483	0.376	0.349
	e_y *	0.900	1.265	1.525	1.025	0.900	0.900	1.025	1.150
	c_x	1.079	1.079	1.0	1.079	1.079	1.079	1.079	1.079
	Z	2221	2020	2221	2221	2221	2221	2221	2221
Perfect	$N_{x_{c\ell}}$	2700	2700	2700	2700	2700	2700	2700	2700
Perfect	m	4	6	6	4	4	3	5	6
Perfect	n	9	8	7	9	9	9	8	8
Imp. A	$N_{x_{cr}}$	2215	2175	2275	2215	2175	2306	2145	2215
Imp. A	n	9	8	7	9	9	9	8	8
Imp. A	$\frac{N_{x_{cr}}}{2700}$	0.8204	0.8055	0.8426	0.8204	0.8055	0.8541	0.7944	0.8204
Imp. B	$N_{x_{cr}}$	2085	2207	2306	2085	2060	2045	2145	2260
Imp. B	n	9	8	7	9	9	8	8	8
Imp. B	$\frac{N_{x_{cr}}}{2700}$	0.7722	0.8174	0.8541	0.7722	0.7630	0.7574	0.7944	0.8170

Point	TS 1f	TS 1g	RS 4a	RS 4b	RS 4c	RS 4d	RS 4e
W, lb.	482	478	490	488	502	500	408
h, in.	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$\bar{\alpha}_x$	12	12	15	17	12	16	13
$\bar{\alpha}_y$	50	45	45	40	55	35	40
λ_{xx}	0.4305	0.4406	0.6419	0.6396	0.4329	0.6297	0.4997
λ_{yy}	0.2736	0.2497	0.0866	0.0581	0.3349	0.1320	0.2876
ρ_{xx} *	61.99	63.45	144.42	184.85	62.33	161.20	84.46
ρ_{yy} *	683.88	505.68	175.28	134.50	1.013.13	161.66	460.14
e_x *	0.349	0.365	0.400	0.450	0.325	0.425	0.350
e_y *	1.275	1.150	1.150	1.025	1.400	0.900	1.025
C_x	1.079	1.135	1.0	1.0	1.0	1.0	1.0
Z	2221	2221	2221	2221	2221	2221	2221
Perfect $N_{x_{c\ell}}$	2700	2700	2700	2700	2700	2700	2700
Perfect m	6	5	4	3	6	4	5
Perfect n	8	8	9	9	7	9	8
Imp. A $N_{x_{cr}}$	2245	2175	2195	2270	2295	2215	2175
Imp. A n	8	8	9	10	7	9	8
Imp. A $\frac{N_{x_{cr}}}{2700}$	0.8315	0.8055	0.8130	0.8407	0.8500	0.8204	0.8055
Imp. B $N_{x_{cr}}$	2290	2175	2075	2045	2290	2085	2175
Imp. B n	8	8	9	8	7	9	8
Imp. B $\frac{N_{x_{cr}}}{2700}$	0.8481	0.8055	0.7685	0.7574	0.8481	0.7722	0.8055

*The ρ's and e's can be found from the $\bar{\alpha}$'s, λ's and C_x's. (see Ref. 23).

Chapter VII

RECOMMENDED DESIGN PROCEDURE FOR AXIALLY LOADED IMPERFECT STIFFENED CYLINDERS

As is seen from the discussion of the results of the two design cases considered, there are no general trends and conclusions that can be applied to all design problems. In addition, the entire study (presented herein) and the resulting conclusions, as far as the position of the optimum design (including the effect of geometric imperfection) in the design space is concerned, are based on the following conjecture: as the applied load $\bar{N}_{xx}$ (including a knockdown factor - used in the linear optimization procedure of References 18 and 23) experiences small changes the corresponding linear optimum point shifts smoothly and by small amounts in the design space. For example, if for a given problem the value of $\bar{N}_{xx}$ changes from 800 to 880 (10% increase) the corresponding optimum design is expected to change from Z = 35,000, $\bar{\alpha}_x = 20$, $\bar{\alpha}_y = 90$, $\lambda_{xx} = 0.64$ and $\lambda_{yy} = 0.20$ to Z = 35,000 + ΔZ, $\bar{\alpha}_x = 20 + \Delta\bar{\alpha}_x$, etc. where the deltas denote changes of less than 10% from the previous values. To further clarify this point and its implications consider the Case 1 design.

First of all, the optimum (linear optimization) design is characterized by (point 2 of Table 4) h = 0.0221 in., $\bar{\alpha}_x = 20$, $\bar{\alpha}_y = 95$, $\bar{\lambda}_{xx} = 0.6319$ and $\lambda_{yy} = 0.2049$ with a weight equal to 755 lbs. The value of the applied load used is 800 lbs/in. Since, at this geometry, the knockdown factor is 0.9125, then the shell is in reality, designed to carry safely, with minimum weight, a load of 730 lbs/in. ($\bar{N}_{xx}$ = [730/knockdown factor] = 800).

Realizing that the knockdown factor is not known apriori and observing that this factor depends on the particular point in the design space, then different values of $\bar{N}_{xx}$ must be considered in the entire optimization procedure, before one is able to zero in to the final optimum design point.

Because of this, the following procedure is suggested in order to achieve the minimum weight design for a given applied load (for the sake of argument, say 1000 lbs/in.) in the presence of geometric imperfections.

a) Specify the maximum allowable amplitude of the geometric imperfection.

b) Guess the value for the knockdown factor (say 0.8 for the ensuing discussion).

c) Employ the design procedure of References 18 and 23 with $\bar{N}_{xx}$ = (1000/0.8 = 1250, and find the minimum weight design point (W, $\bar{\alpha}_x$, $\bar{\alpha}_y$, λ_{xx} and λ_{yy}).

d) If the actual knockdown factor is within 10% from the guessed value of 0.8, then perform comparison studies as outlined herein.

e) Repeat steps c) and d) for values of $\bar{N}_{xx}$ equal to 1000/(0.72) and 1000/(0.88). An examination of the three comparison studies will yield the optimum configuration within any desired (reasonable) accuracy.

f) If the actual knockdown factor differs by more than 10% from the guessed value (say 20% lower), then repeat step c) for values of $\bar{N}_{xx}$ equal to 100/(0.55), 1000/(0.65), and 1000/(0.80) and perform the comparison studies as mentioned in step e).

By following the above procedure, the designer is assured of arriving at a

minimum weight design, which can carry safely a uniform axial compression of 1000 lbs/in.

Appendix A

BUCKLING OF IMPERFECT STIFFENED CYLINDERS UNDER COMBINED AXIAL COMPRESSION AND PRESSURE

The buckling analysis of geometrically imperfect, stiffened, thin, circular, cylindrical shells under uniform axial compression has been presented in Chapters II-IV. The purpose of this Appendix is to show the extension of the analysis for the load cases of hydrostatic pressure, lateral pressure, and combined uniform axial compression with lateral pressure.

Most of the investigations, reported in the open literature which deal with buckling of imperfect configurations, are for uniform axial compression and isotropic, constant thickness geometries. Hutchinson and Amazigo[9] investigated the buckling of imperfect stiffened cylinders (stringer and ring stiffened only) under hydrostatic pressure. There is no reported investigation for stringer and ring stiffened geometries.

The self-equilibrated lateral load p(x,y) [positive inward] is expanded in terms of Fourier series

$$p(x,y) = \sum_{i=0}^{K} \bar{p}_i(x) \cos \frac{iny}{R} \tag{A-1}$$

Then all equations in the body of the report must be changed appropriately to reflect the inclusion of this load. For example, the term $-\bar{p}_o(x)$ must be added to Eq. (26a), the term $-\bar{p}_i(x)$ to Eq. (26b), and the following term must be added to the total potential expression, Eq. (27).

$$-\pi R \int_0^L \left[2\bar{p}_o(x)\, W_o(x) + \sum_{i=1}^{K} \bar{p}_i(x)\, W_i(x) \right] dx$$

The related computer program (Appendix B) includes these changes.

A number of illustrative examples are considered and the results are presented in tabular form (see Table A-1) for uniform pressure only. For all examples, the boundary conditions are taken to be the classical simply supported ones, SS3. The program, though, is written for all types of lateral (clamped, simply supported, free, etc.) as well as in-plane boundary conditions. Some of the examples considered serve as bench marks for the developed methodology and computer program. In addition, new results are reported. For all cases, two types of geometric imperfection are considered which are characterized by $\xi = 1$ and $\xi = 2$. These are:

$$\xi = 1 \qquad w^o(x,y) = t \sin \frac{m\pi x}{L} + 0.1t \sin \frac{m\pi x}{L} \cos \frac{ny}{R} \tag{A-2}$$

$$\xi = 2 \qquad w^o(x,y) = t \sin \frac{m\pi x}{L} \cos \frac{ny}{R} \tag{A-3}$$

Note that the imperfection characterized by Eq. (A-2) virtually denotes an axisymmetric type of imperfection, while the one characterized by Eq. (A-3) denotes a symmetric type of imperfection. In both cases the amplitude is approximately one skin thickness. The values of m and n that correspond to the most sensitive geometry are reported in Table A-1.

The examples reported have the following common geometry

$$\begin{aligned} &L = 4''; \quad R = 4''; \quad t = 0.04'' \\ &\nu = 0.3; \quad E = 10.5 \times 10^6 \text{ psi}; \quad Z = 95.3 \end{aligned} \tag{A-4}$$

Note that examples 1 and 2 correspond to internal ring stiffening only, examples 3 and 4 to internal stringer stiffening only, examples 5, 6, 8-11 to internal ring and stringer stiffening, and example 7 to external ring and stringer stiffening. In addition, the loading for all the examples is lateral pressure for examples 1, 3, and 5, hydrostatic pressure for examples 2, 4, 6, and 7, axial compression for example 8, and combined loading for examples 9-11 (internal pressure of half and one atmosphere respectively for examples 9 and 10, and external pressure of one atmosphere for example 11).

Examples 2 and 4 are taken from Ref. 9 and the results are in very good agreement.

Among the important observations, on the basis of the generated data, one may list the following:

1) For pressure loading the ring only stiffened cylinder is sensitive to geometric imperfections while the stringer only stiffened cylinder is not. This observation is true for lateral pressure ($\bar{N}_{xx} = 0$; examples 1 and 3) as well as hydrostatic pressure ($\bar{N}_{xx} = pR/2$; examples 2 and 4). Furthermore, the presence of the axial load in the case of hydrostatic pressure (compare results of examples 1 to 2 and 3 to 4) aggravates the imperfection sensitivity.
2) When stiffening in both directions is used the configuration is still sensitive but not as sensitive as the ring only configuration (compare the results of examples 5 to 1 and 6 to 2).
3) It is concluded, in Ref. 9, that positioning the rings on the outside (rings only configuration) makes the shell more sensitive

to geometric imperfections as compared to internal positioning of the rings. This observation is also true for stringer and ring stiffened cylinders (compare the results of example 7 to 6).

4) For stiffened cylinders (both rings and stringers) under uniform axial compression with or without lateral pressure the presence of internal or external pressure of up to one atmosphere does not have an appreciable effect on the final results. Linear theory predicts minimal effect, in the right direction (examples 8-11) (negative p implies internal pressure) and nonlinear theory shows that, for the same imperfection, the sensitivity is not affected by the presence of the small internal or external pressure.

5) The buckled shape in the circumferential direction (n) is virtually the same for the nonlinear theory in the presence of geometric imperfections, as it is for the perfect geometry linear theory.

Finally, note that all results are derived for one value of the curvature parameter Z and two stiffener geometries; therefore, no generality should be attached to the above observations. If one is interested in performing extensive parametric studies in order to draw general conclusions the present computer program is very efficient (Appendix B).

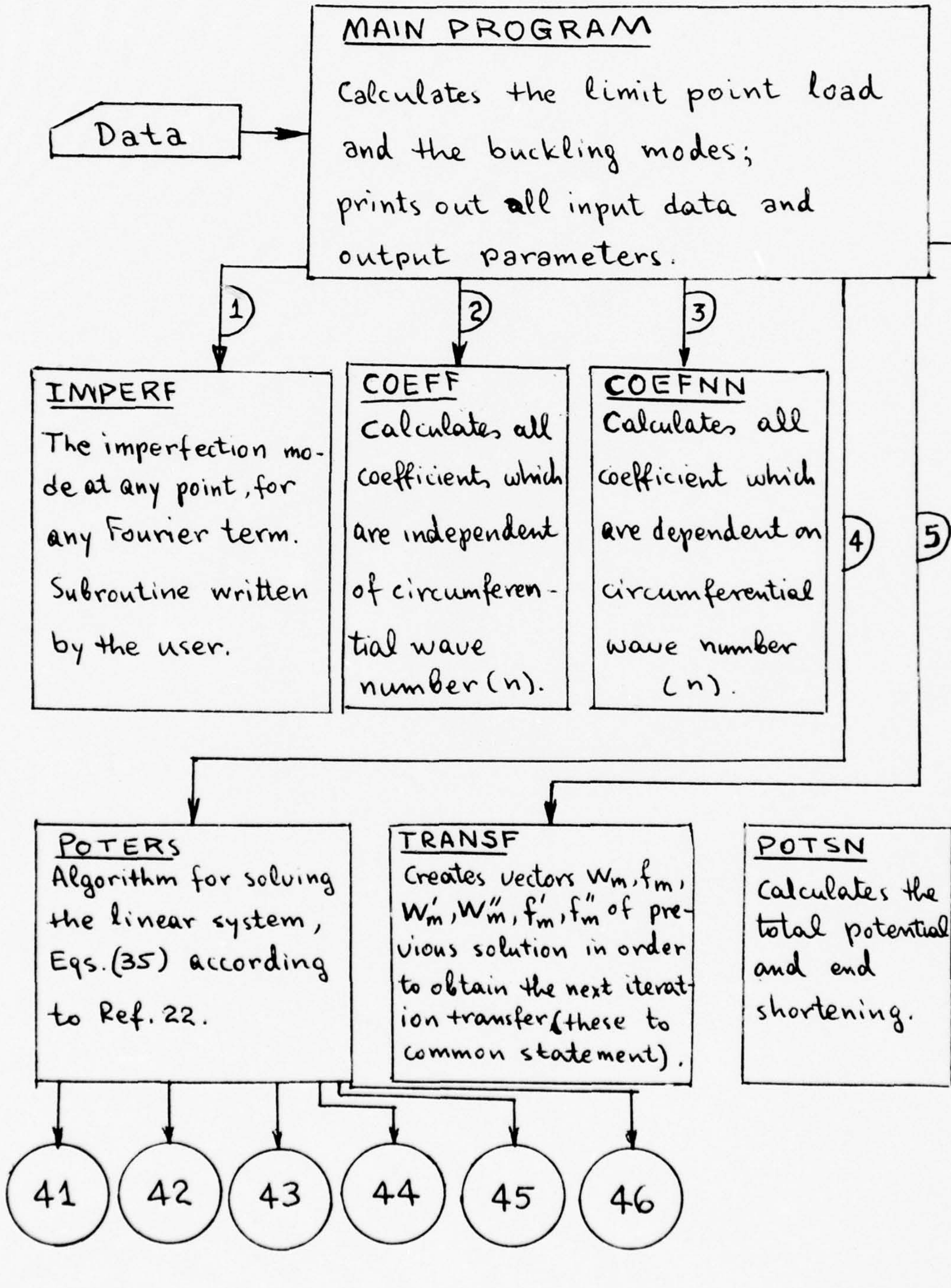
Table A-1. Buckling Results of Imperfect Stiffened Cylinders

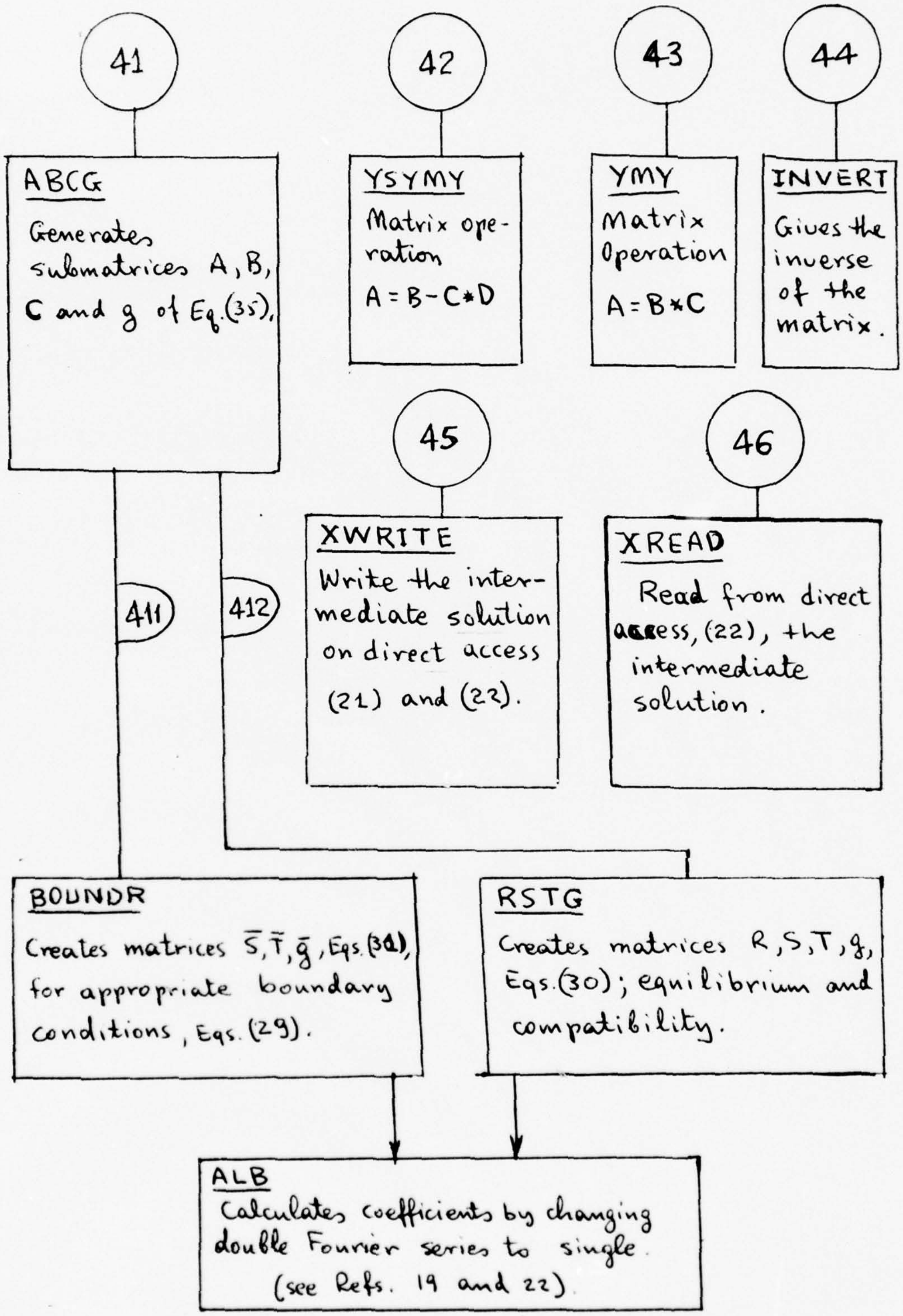
Example Number	Loading	Geometry						Classical (Linear) Solution				Imperfection Parameter	Nonlinear Solution			
		λ_{xx}	λ_{yy}	e_x/t	e_y/t	ρ_x	ρ_y	$N_{xx_{cr}}$	p_{cr}	n	m	ξ	$N_{xx_{cr}}$	p_{cr}	$\frac{N_{xx_{cr}} \text{ or } p_{cr}}{\text{Classical}}$	n
1	$\bar{N}_{xx} = 0$, p	0	0.91	0	6	0	100	0	4827	3	1	1	0	3800 - 4000	0.787 - 0.829	3
2	$\bar{N}_{xx} = \frac{pR}{2}$, p	0	0.91	0	6	0	100	2619	1309	4	6	1	1500 - 1600	750 - 800	0.573 - 0.611	3
3	$\bar{N}_{xx} = 0$, p	0.91	0	6	0	100	0	0	399	13	1	1	0	550 - 600	1.378 - 1.504	22
4	$\bar{N}_{xx} = \frac{pR}{2}$, p	0.91	0	6	0	100	0	776	388	13	1	1	900 - 1000	450 - 500	1.160 - 1.290	17
5	$\bar{N}_{xx} = 0$, p	0.91	0.91	6	6	100	100	0	7060	4	1	2	0	6000 - 6500	0.850 - 0.920	4
6	$\bar{N}_{xx} = \frac{pR}{2}$, p	0.91	0.91	6	6	100	100	9686	4843	3	1	2	8500 - 9000	4250 - 4500	0.878 - 0.929	3
7	$\bar{N}_{xx} = \frac{pR}{2}$, p	0.91	0.91	-6	-6	100	100	16200	8100	4	1	2	9000 - 10000	4500 - 5000	0.555 - 0.617	4
8	p = 0, $\bar{N}_{xx}$	0.91	0.455	6	3	100	20	19790	0	4	1	2	15250 - 15630	0	0.771 - 0.790	4
9	p = -7.2, N_{xx}	0.91	0.455	6	3	100	20	19830	-7.2	4	1	2	15000 - 15500	-7.2	0.756 - 0.782	4
10	p = -14.4, N_{xx}	0.91	0.455	6	3	100	20	19880	-14.4	4	1	2	15000 - 15500	-14.4	0.755 - 0.780	4
11	p = 14.4, N_{xx}	0.91	0.455	6	3	100	20	19690	14.4	4	1	2	15000 - 15500	14.4	0.762 - 0.787	4

APPENDIX B

COMPUTER PROGRAM

I. BLOCK DIAGRAM




41
42
43
44
ABCG
Generates submatrices A, B, C and g of Eq.(35).
YSYMY
Matrix operation
A = B − C*D
YMY
Matrix Operation
A = B*C
INVERT
Gives the inverse of the matrix.
45
46
XWRITE
Write the intermediate solution on direct access (21) and (22).
XREAD
Read from direct access, (22), the intermediate solution.
411
412
BOUNDR
Creates matrices $\bar{S}, \bar{T}, \bar{g}$, Eqs.(31), for appropriate boundary conditions, Eqs. (29).
RSTG
Creates matrices R, S, T, g, Eqs.(30); equilibrium and compatibility.
ALB
Calculates coefficients by changing double Fourier series to single. (see Refs. 19 and 22)

COMMON CARDS

1) Common/CINTG/NEQPOT, MI (500)

NEQPOT - Number of points in axial direction

MI (500) - The order of Eq. I, MI(I) [according to Ref. 22]

2) COMMON/BOUND/LS1, LSN

Definition of boundary condition at the first point (LS1), and at the last point (LSN) of the shell.

3) Common/FIDFR/DELTA, AL1, GA1, AL2, BT2, GA2.

Coefficients of finite difference form, Δ, $\alpha^1 = -1/2\Delta$, $\gamma^1 = 1/2\Delta$, $\alpha^2 = \gamma^2 = 1/\Delta^2$, $\beta^2 = -2/\Delta^2$.

4) COMMON/FOURIR/KFOUR, K6, K4, K3, K2, K1.

Fourier series limit (K=KFOUR) and parameters dependent on K.

5) COMMON/GEOM/RR, DD, H_{11}, H_{12}, H_{22}, Q_{11}, Q_{12}, Q_{22}, D_{11}, D_{12}, D_{22}

Shell geometric parameters; R, D, h_{ij}, q_{ij}, d_{ij} [Eqs. (9)].

6) COMMON/FACTOR/C1, C2, ... C12.

Coefficients which are dependent on circumferential wave number.

7) COMMON/FACT2/DL1-DL4, DA1-DA4, DB2, DB3, DB4, XNI, Exxp.

Coefficients $\gamma_1, \gamma_2, \gamma_3, \gamma_4, a_1, a_2, a_3, a_4, b_2, b_3, b_4, \nu$, Exxp.

8) COMMON/CDISK/I21(501),I22(501)

Direct access data set 21 and 22.

9) COMMON/FACT3/DLS,XL,XH.

parameter, XL=L, XH=t.

10) COMMON/PRES1/WM(200,5),ETM(200,5),WMP(200,5)

The vector of previous solution W_m, W_m'', and W_m' at point $\underline{l}$ for Fourier term $\underline{i}$.

11) COMMON/PRES2/WZ(200,5),WZP(200,5),WZPP(200,5)

Imperfection mode W^o, $W^{o\prime}$, $W^{o\prime\prime}$ at point $\underline{l}$ for Fourier term $\underline{i}$.

12) COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)

The vector of previous solution f_m, f_m'', and f_m' at point $\underline{l}$ for Fourier term $\underline{i}$.

13) COMMON/XXLOAD/XPRES

XPRES - hydrostatic pressure.

II. FLOW CHART FOR DATA PREPARATION

START

TITLE — 9A8

Any alphanumeric information for case title (72 columns).

10I6

NEQPOT, KFOUR, LS1, LSN, LPRINT, LMOD, IDDET.

NEQPOT : No. of points in x-direction, ($5 \le$ NEQPOT ≤ 200).

KFOUR : FOURIER SERIES LIMIT (K), ($0 \le$ KFOUR ≤ 4).

IT IS POSSIBLE TO INCREASE KFOUR AND NEQPOT BY CHANGING THE DIMENSION AND COMMON STATEMENT.

LS1 : BOUNDARY CONDITION OF THE FIRST POINT.

LSN : BOUNDARY CONDITION OF THE LAST POINT.

Note: LS=1 for SS1, LS=2 for SS2, LS=3 for SS3
LS=4 for SS4, LS=5 for CC1, LS=6 for CC2
LS=7 for CC3, LS=8 for CC4, LS=9 FF1,
LS=10 for symmetry condition, and LS=11 for antisymmetry condition.

(cont'd)

A1

A1

(cont'd)

LPRINT 0 = minimum printout ; 1 = maximum printout

LMOD 0 : does not print modes ; 1 = prints modes

IDDET 0 : does not calculate determinant

1 : calculates determinant and prints it.

AP (12K+4,12K+4), BP (12K+4,12K+4), CP (12K+4,12K+4)
PR (12K+4,12K+4), GP (12K+4,1), XP (12K+4,1), T1 (12K+4), C (12K+4),
MT (12K+4), V1 ((12K+4)*(12K+4)).

COMMON/PRES1/WM (NEQPOT,K+1), ETA (NEQPOT,K+1), WMP (NEQPOT,K+1).

COMMON/PRES2/WZ (NEQPOT,K+1), WZP (NEQPOT,K+1), WZPP (NEQPOT,K+1).

COMMON/PRES3/FM (NEQPOT,2K), XFM (NEQPOT,2K), FMP (NEQPOT,2K).

RR, XL, XH, ELAS, XNI 6E12.4

RR : Radius of the cylinder
XL : Length of the cylinder
XH : Thickness of the cylinder
ELAS : Modulus of Elasticity
XNI : Poisson's ratio.

A2

A2

XLAMD, YLAMD, EX, EY, RHOX, RHOY

6E12.4

XLAMD $= \lambda_{xx} = (1-\nu^2) A_x / t \ell_x$

YLAMD $= \lambda_{yy} = (1-\nu^2) A_y / t \ell_y$

EX = Stringer eccentricity parameter (positive inward)

EY = Ring eccentricity parameter (positive inward)

RHOX $= \rho_{xx} = EI_{x_c} / D \ell_x$

RHOY $= \rho_{yy} = EI_{y_c} / D \ell_y$

The user must write subroutine IMPERF for the definition of the imperfection and the derivatives.

WZ (I,J) $= W^0$ (positive inward)

WZP(I,J) $= W^{0\prime}$

WZPP(I,J) $= W^{0\prime\prime}$

I = 1, NEQPOT mesh point

J = 1, KFOUR+1 for all Fourier terms

Note that since $W^0 = \sum_{i=0}^{K} W_i^0(x) \cos \frac{iny}{R}$

then

J = 1 for $i=0$, and J = KFOUR+1 for i = KFOUR

A3

A3

6E12.4

DLND, XNXX, XPRESS, DNXP, ACCUR, RI

DLND = 1	For fixed lateral pressure p find $N_{x_{cr}}$.
DLND = 2	For fixed axial compression $\bar{N}_{xx}$, find p_{cr}.
DLND = 3	Axial load and pressure are related by the factor XNXX (N_x = XNXX * XPRES)
XNXX	For DLND = 1 is the initial axial load For DLND = 2 is the fixed axial load. For DLND = 3 is the factor that relates N_{xx} and p (positive XNXX = compression).
XPRES	For DLND = 1 is the fixed pressure For DLND = 2,3 is the initial pressure (positive inward)
DNXP	For DLND = 1 is the increment in axial load For DLND = 2,3 is the increment in pressure
ACCUR	The required load accuracy in percent
RI	Maximum number of load points

* N_x = XNXX + RI * DNXP (for DLND = 1)
  p = XPRES + RI * DNXP (for DLND = 2,3).

A4

A4

INON

1016

INON = 1 The initial solution for the nonlinear system comes from the linear solution

INON = 2 The initial solution for the nonlinear system (XNXX + DNXP or XPRES + DNXP) is the solution at the previous load level (i.e. XNXX or XPRES).

(It is desirable to take INON = 1.)

A5

A5

NNN , LNNN , ILNW

1016

NNN – The circumferential wave number, n. The program finds the limit point for this n.

LNNN = 0 : The program does which n gives the minimum potential energy.

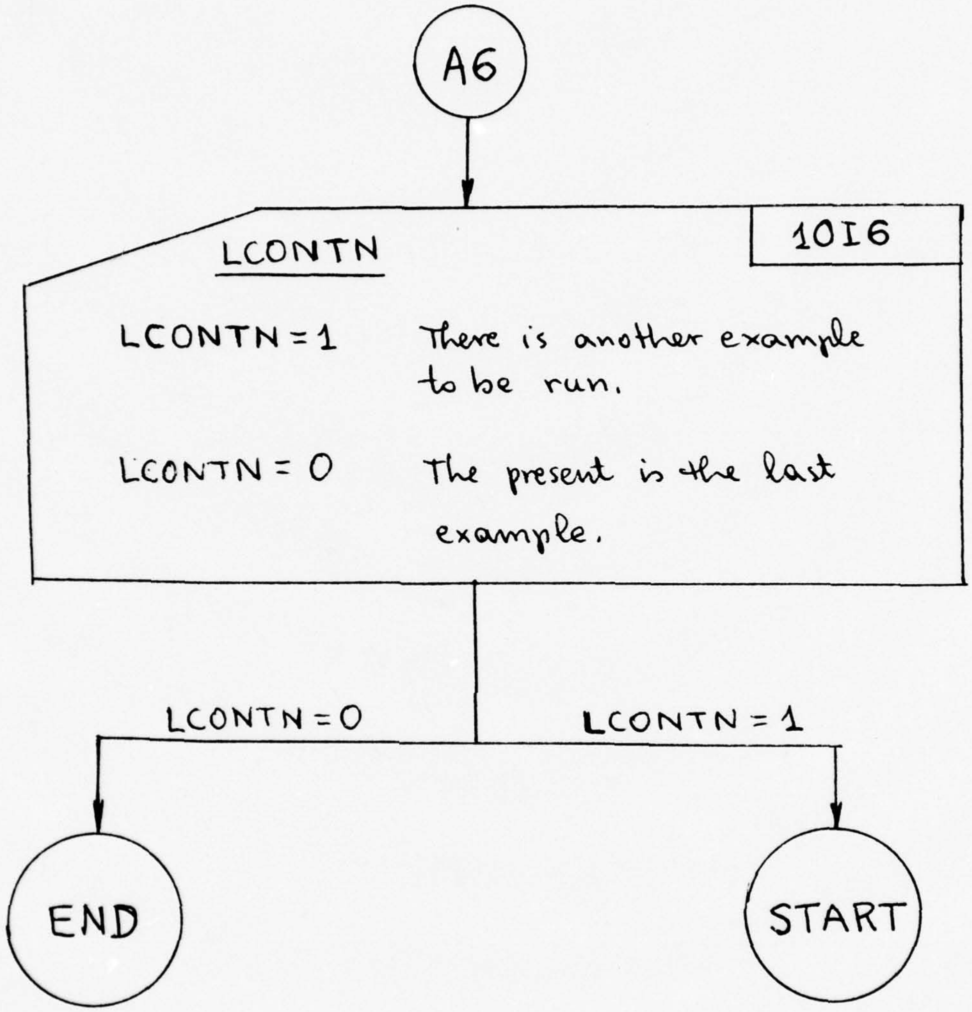
LNNN = 1 The program finds the limit point for NNN, and calculates the total potential for values surounding NNN at load levels lower than limit point (NNN).

LNNN = 2 The program calculates the total potential only for the given load XNXX. In this case set XNXX close to the estimated limit point.

ILNW ≤ 10 Ten is the maximum number of n-values for which the program calculates the total potential in order to find the minimum (and corresponding n).

For example, if $n=5$ and ILNW = 2 the program calculates the total potential for $n = 5 ; 6$. If $U_T(n=6) < U_T(n=5)$ then it calculates U_T for $n=7$, if not for $n=4$.

A6


A6
LCONTN
1OI6
LCONTN = 1
There is another example to be run.
LCONTN = 0
The present is the last example.
LCONTN = 0
LCONTN = 1
END
START

III. COMPUTER PROGRAM

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE21,TAPE21
     1,TAPE22)
C     POST BUCKLING OF STIFFENED CYLINDRICAL SHELLS UNDER UNIFORM AXIAL
C    COMPRESSION (NONLINEAR THEORY)
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/BOUND/LS1,LSN
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/CDISK/I21(501),I22(501)
      COMMON/FACT3/DL5,XL,XH
      COMMON/PRES1/WM(200,5),ETM(200,5),WMP(200,5)                      **
      COMMON/PRES2/WZ(200,5),WZP(200,5),WZPP(200,5)                     **
      COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)                      **
      COMMON/XXLOAD/XPRES
      DIMENSION WWM(5),FFM(8)                                           **
      DIMENSION TI(10)
      DIMENSION WF(2,5),XWF(2,5),FF(2,8),XFF(2,8)                       **
      DIMENSION AP(52,52),BP(52,52),CP(52,52),PR(52,52),GP(52,1)        **
      DIMENSION XP(52,1),T1(52),CC(52),MT(52),V1(2704)                  **
C     2704=52*52
      DIMENSION WCON(20,5),FCON(20,8)                                   **
C     ALL THE CARDS WITH SIGN   **  IN COLUMNS 73,74  DEPEND ON  NUMBER
C OF POINTS AND  KFOUR
      EQUIVALENCE (AP(1,1),V1(1))
      CALL OPENMS(21,I21,501,0)
      CALL OPENMS(22,I22,501,0)
      ECONV=0.000001
      MAXN=52                                                           **
      MAX2=MAXN*MAXN
      NRHS=1
      NJ=200                                                            **
      NW=5                                                              **
      NF=8                                                              **
C    NJ,NW,NF - FOR DIMENSION  --  NJ=MAXIMUM POINTS IN AXIAL DIRECTION
C  NW= MAXIMUM KFOUR+1    ,    NF= MAXIMUM 2*KFOUR   ,   MAXN=12*KFOUR+4
C IN ORDER TO INCREASE THE CAPABILITY OF THE PROGRAM FOR MANY POINTS IN AXIAL
C  DIRECTION AND HIGHER LIMIT OF FOURIER SERIES THE USER HAS TO CHANGE
C  ALL THE CARDS WITH THE SIGN  **  IN COLUMNS  73 AND  74
 1111 WRITE(6,20)
      READ(5,10)(TI(I),I=1,9)
      WRITE(6,60)
      WRITE(6,10)(TI(I),I=1,9)
      READ(5,100)NEQPOT,KFOUR,LS1,LSN,LPRINT,LMOD,IDCET
      IF(LPRINT.EQ.1)LMOD=1
      READ(5,200)RR,XL,XH,ELAS,XNI
      READ(5,200)XLAMD,YLAMD,EXX,EYY,RHOX,RHOY
      EX=-EXX
      EY=-EYY
C     *************
        CALL COEFF(EX,EY,XLAMD,YLAMD,RHOX,RHOY,ELAS)
C  **************
      WRITE(6,300)NEQPOT,KFOUR,LS1,LSN
      WRITE(6,400)RR,XL,XH,ELAS,XNI,DD,EXXP
      WRITE(6,573)XLAMD,YLAMD,EXX,EYY,RHOX,RHOY
```

```
C     ***********
        CALL IMPERF
C     ***************
        WRITE(6,508)
        DO 85 IK=1,K1
        LK=IK-1
        WRITE(6,510)LK
        WRITE(6,520)
        XX=0.
        DO 85 I1=1,NEQPOT
        WRITE(6,509)I1,XX,WZ(I1,IK),WZP(I1,IK),WZPP(I1,IK)
        XX=XX+DELTA
   85   CONTINUE
        IF (LPRINT.NE.1) GO TO 39
        WRITE(6,500)DELTA,AL1,GA1,AL2,BT2,GA2
        WRITE(6,501)H11,H12,H22,Q11,Q12,Q22
        WRITE(6,502)D11,D12,D22,DB2,DB3,DB4
        WRITE(6,503)DL1,DL2,DL3,DL4,DL5
        WRITE(6,504)DA1,DA2,DA3,DA4
   39   CONTINUE
        DO 63 I1=1,NEQPOT
        DO 64 J1=1,K1
        WM(I1,J1)=0.
        ETM(I1,J1)=0.
        WMP(I1,J1)=0.
   64   CONTINUE
        DO 65 J1=1,K2
        FM(I1,J1)=0.
        XFM(I1,J1)=0.
        FMP(I1,J1)=0.
   65   CONTINUE
   63   CONTINUE
        READ(5,200)DLND,XNXX,XPREE,DNXX,ACCUR,RII
        IRR=RII
        IF(IRR.EQ.0)IRR=1
        IDLND=DLND
        DNX=DNXX
        XPRES=XPREE
        XFNX=XNXX
C XFNX=AXIAL COMPRESSION, XPRES=HYDROSTATIC PRESSURE, XNX=EITHER XFNX
C OR XPRES ACCORDING TO IDLND
        GO TO(71,72,73),IDLND
   71   WRITE(6,511)XPRES,XFNX,DNX,ACCUR
        XNX=XNXX
        XN11=XNXX
        GO TO 74
   72   WRITE(6,512)XFNX,XPRES,DNX,ACCUR
        XNX=XPREE
        XN11=XPRES
        GO TO 74
   73   WRITE(6,513)XNXX,XPRES,DNX,ACCUR
        TLMDX=XNXX
        XFNX=TLMDX*XPRES
        XNX=XPREE
        XN11=XPRES
   74   TXNX=1000.*XNX
        READ(5,100)IMON
```

3

```
      READ(5,100)NNN,LNNN,ILNW
      NWAVE=NNN
C   *********************
      CALL CCEFNN(NWAVE)
C    *********************
      WRITE(6,505)NNN
      IF(LPRINT.NE.1) GO TO 49
      WRITE(6,506)C1,C2,C3,C4,C5,C6
      WRITE(6,507)C7,C8,C9,C10,C11,C12
   49 CONTINUE
      ILR=0
      LICON=1
      IPOTT=0
      CALL SECOND(TIM1)
      WRITE(6,793)TIM1
      TIM2=TIM1
      TIM4=TIM2
      IINN=0
  555 LN=1
      IF(IDLND.EQ.1)XFNX=XNX
      IF(IDLND.EQ.2.OR.IDLND.EQ.3)XPRES=XNX
      IF(IDLND.EQ.3)XFNX=TLMDX*XPRES
      IDET=ICDET
      CALL FCTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,CC,MT,T1,V1,MAX2,
     1IXPM,CETM,XFNX,LN,NJ,NK,NF)
      IF(LPRINT.NE.1) GO TO 101
      CALL SECOND(TIM3)
      TIM1=TIM3-TIM2
      TIM2=TIM3
      TIM4=TIM3
      WRITE(6,201)NWAVE,XFNX,XPRES,TIM1
  101 CALL TRANSF(WF,XWF,FF,XFF,NW,NF,2,T1,MAXN,1,LPRINT)
  444 IDET=ICDET
      IF(IDLND.EQ.1)XFNX=XNX
      IF(IDLND.EQ.2.OR.IDLND.EQ.3)XPRES=XNX
      IF(IDLND.EQ.3)XFNX=TLMDX*XPRES
      IMAX=1
      WMAX=0.
      ITER=0
      DO 102 J1=1,K1
  102 WMAX=WMAX+WM(1,J1)
      DO 103 I1=2,NEQPCT
      WMM=0.
      DO 104 J1=1,K1
  104 WMM=WMM+WM(I1,J1)
      IF(ABS(WMM).LE.ABS(WMAX)) GO TO 103
      WMAX=WMM
      IMAX=I1
  103 CONTINUE
      JWMAX=1
      WWM(1)=WM(IMAX,1)
      AWWM=WWM(1)
      IF(K1.EQ.1) GO TO 1051
      DO 105 J1=2,K1
      WWM(J1)=WM(IMAX,J1)
      IF(ABS(WWM(J1)).LE.ABS(AWWM)) GO TO 105
      AWWM=WWM(J1)
```

```
      JWMAX=J1
  105 CONTINUE
 1051 JFMAX=1
      FFM(1)=FM(IMAX,1)
      AFFM=FFM(1)
      IF(K2.EQ.1) GO TO 333
      DO 106 J1=2,K2
      FFM(J1)=FM(IMAX,J1)
      IF(ABS(FFM(J1)).LE.ABS(AFFM)) GO TO 106
      AFFM=FFM(J1)
      JFMAX=J1
  106 CONTINUE
  333 LN=2
      ITER=ITER+1
      IF(ITER.LE.10) GO TO 113
      WRITE(6,114)ITER
      GO TO 9999
  113 CALL FCTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,CC,MT,T1,V1,MAX2,
     1IXPM,DETM,XFNX,LN,NJ,NW,NF)
      IF(LPRINT.NE.1) GO TO 111
      CALL SECOND(TIM3)
      TIM1=TIM3-TIM2
      TIM2=TIM3
      WRITE(6,112)ITER,NWAVE,XFNX,XPRES,TIM1
  111 CALL TRANSF(WF,XWF,FF,XFF,NW,NF,2,T1,MAXN,1,LPRINT)
      DO 115 J1=1,K1
      IF(WM(IMAX,J1).NE.0.) GO TO 57
      WCON(ITER,J1)=0.
      GO TO 115
   57 CONTINUE
      WCON(ITER,J1)=ABS((WM(IMAX,J1)-WWM(J1))/WM(IMAX,J1))
  115 CONTINUE
      WCH=WCON(ITER,JWMAX)
      IWH=JWMAX
      DO 116 J1=1,K2
      IF(FM(IMAX,J1).NE.0.) GO TO 58
      FCON(ITER,J1)=0.
      GO TO 116
   58 CONTINUE
      FCON(ITER,J1)=ABS((FM(IMAX,J1)-FFM(J1))/FM(IMAX,J1))
  116 CONTINUE
      FCH=FCON(ITER,JFMAX)
      IFH=JFMAX
      IF(LPRINT.NE.1) GO TO 117
      WRITE(6,118)ITER,WCH,FCH
      WRITE(6,119)(J1,WCON(ITER,J1),J1=1,K1)
      WRITE(6,119)(J1,FCON(ITER,J1),J1=1,K2)
  117 IF(WCH.GT.ECONV) GO TO 194
      IF(FCH.GT.ECONV) GO TO 194
      GO TO 195
  194 IF(ITER.LE.2) GO TO 196
      IF(WCON(ITER,IWH).GT.WCON(ITER-1,IWH)) GO TO 197
      IF(FCON(ITER,IFH).GT.FCON(ITER-1,IFH)) GO TO 197
      GO TO 196
  197 IF(XNX.NE.XN11) GO TO 198
      WRITE(6,991)XNX
      GO TO 9999
```

```
196 DO 131 J1=1,K1
131 WWM(J1)=WM(IMAX,J1)
    DO 132 J1=1,K2
132 FFM(J1)=FM(IMAX,J1)
    GO TO 333
195 IF(IDLND.EQ.1)XFNX=XNX
    IF(IDLND.EQ.2.OR.IDLND.EQ.3)XFRES=XNX
    IF(IDLND.EQ.3)XFNX=TLMCX*XPRES
    CALL FCTSN(FCT,STRY,STRA,1,1,1,XFNX)
    CALL SECOND(TIM3)
    TIM1=TIM3-TIM4
    TIM2=TIM3
    TIM4=TIM3
    WRITE(6,241)XFNX,XFRES,NWAVE,ITER,TIM1
    WRITE(6,242)POT,STRY,STRA
    IF(IDET.EQ.1) WRITE(6,243)DETM,IXPM
    IF(LMCD.NE.1) GO TO 476
    CALL TRANSF(WF,XWF,FF,XFF,NW,NF,2,T1,MAXN,2,3)
476 CONTINUE
    IF(LNNN.EQ.2.AND.LICON.NE.10) GO TO 566
    IF(LICON.NE.10) GO TO 629
    ILR=1
    GO TO 777
629 XNX1=XNX
    IINN=IINN+1
    IF(IINN.LE.IRR)GO TO 721
    WRITE(6,722)IINN
    GO TO 9999
721 CONTINUE
    XNX=XNX+DNX
    IF(TXNX.GT.XNX) GO TO 244
    DNX=DNX/2.
    XNX=XNX-DNX
    ADN=DNX*100/XNX
    IINN=IINN-1
    IF(ADN.GT.ACCUR) GO TO 244
    XNX=XNX1
    GO TO 819
244 IF(INCN.EQ.1) GO TO 555
    REWIND 20
    WRITE(20)((WM(I1,J1),J1=1,K1),I1=1,NEQPOT),((ETM(I1,J1),J1=1,K1)
   1,I1=1,NEQPOT),((WMP(I1,J1),J1=1,K1),I1=1,NEQPOT),((FM(I1,J1)
   2,J1=1,K2),I1=1,NEQPOT),((XFM(I1,J1),J1=1,K2),I1=1,NEQPOT),
   3((FMP(I1,J1),J1=1,K2),I1=1,NEQPOT)
    GO TO 444
198 IF(LICON.NE.10) GO TO 429
    ILR=0
    GO TO 777
429 IF(LNNN.EQ.0) GO TO 249
    IPOTT=IPOTT+1
    IF(IPOTT.GT.1) GO TO 249
    POTT=FCT
    FXNX=XNX-DNX
249 ADN=DNX*100./XNX
    WRITE(6,545)NWAVE,XNX
    TXNX=XNX
    XNX=XNX-DNX
```

6

```
      IINN=0
      IF(ADN.LE.ACCUR) GO TO 819
      DNX=DNX/2.
      XNX=XNX+DNX
      IF(INCN.EQ.1) GO TO 555
      REWIND 20
      READ(20)   ((WM(I1,J1),J1=1,K1),I1=1,NEQPOT),((ETM(I1,J1),J1=1,K1)
     1,I1=1,NEQPOT),((WMF(I1,J1),J1=1,K1),I1=1,NEQPOT),((FM(I1,J1)
     2,J1=1,K2),I1=1,NEQPOT),((XFM(I1,J1),J1=1,K2),I1=1,NEQPOT),
     2((FMF(I1,J1),J1=1,K2),I1=1,NEQPOT)
      GO TO 444
  819 IF(IDLND.EQ.1) XFNX=XNX
      IF(IDLND.EQ.2.OR.IDLND.EQ.3)XPRES=XNX
      IF(IDLND.EQ.3)XFNX=TLMDX*XPRES
      WRITE(6,785)NWAVE,XFNX,XPRES,POT,STRY,STRA
C  CALCULATION OF CRITICAL WAVE NUMBER
  566 WRITE(6,20)
      IF(LNNN.EQ.0) GO TO 9999
      WRITE(6,584)
      ILR=1
      NWPRIN=NWAVE
      NWAVE=NWAVE+1
      IF(LNNN.EQ.2) POTT=POT
      POTMIN=POTT
      NMIN=NNN
      INWAVE=0
      ISTOP=0
      I9=0
      POT=POTT
  777 IF(IDLND.EQ.1)XFNX=XNX
      IF(IDLND.EQ.2.OR.IDLND.EQ.3)XPRES=XNX
      IF(IDLND.EQ.3)XFNX=TLMDX*XPRES
      IF(ILR.EQ.1) GO TO 778
      WRITE(6,582)NWAVE,XFNX,XPRES
      GO TO 9999
  778 INWAVE=INWAVE+1
      IINN=0
      IF(LNNN.EQ.2)PXNX=XNX
      XNX=PXNX
      LICON=10
      IF(INWAVE.EQ.1)     GO TO 391
      NWPRIN=NWAVE
      IF(I9.GE.1) GO TO 694
      IF(POTMIN.LE.POT) GO TO 139
      POTMIN=POT
      NMIN=NWAVE
      NWAVE=NWAVE+1
      GO TO 391
  139 IF(NWAVE.LE.NNN+1) GO TO 549
      ISTOP=1
      GO TO 185
  549 I9=I9+1
      IF(I9.GT.1) GO TO 694
      NWAVE=NNN-1
      GO TO 391
  694 IF(POTMIN.LE.POT) GO TO 695
      POTMIN=POT
```

```
      NMIN=NWAVE
      NWAVE=NWAVE-1
      GO TO 391
  695 ISTOP=1
      GO TO 185
  391 CALL COEFNN(NWAVE)
      CALL SECOND(TIM2)
  185 WRITE(6,581)NWFRIN,PXNX,POT,TIM2
      IF(NWAVE.LE.0) GO TO 9999
      IF(ISTOP.NE.1) GO TO 798
      WRITE(6,789)XNX,FOTMIN,NMIN
      GO TO 9999
  798 IF(INWAVE.GT.ILNW) GO TO 9999
      GO TO 555
 9999 READ(5,100)LCONTN
      IF(LCONTN.EQ.1) GO TO 1111
   20 FORMAT(1H1)
   10 FORMAT(1H0,9A8)
   60 FORMAT(//25H BEGINNING OF NEXT CASE      ///)
  100 FORMAT(10I6)
  200 FORMAT(6E12.4)
  300 FORMAT(//,2X,"NO. OF POINTS=",I8,2X,"KFOUR=",I8,2X,"BOUND.CON OF
     1POINT 1=",I8,2X,"BOUND.CON OF POINT NEGPOT=",I8)
  400 FORMAT(//,2X,"P=",E12.4,2X,"XL=",E12.4,2X,"XH=",E12.4,2X,
     1"ELAS=",E12.4,2X,"XNI=",E12.4,2X,"DD=",E12.4,2X,"EXXP=",E12.4)
  508 FORMAT(//,2X,"THE IMPERFECTION FORM IN AXIAL DIRECTION IS"/)
  510 FORMAT(//,2X,"THE IMPERFECTION FOR CIRCUMFERENTIAL WAVE  ",I6/)
  520 FORMAT(//,4X,"POINT",9X,"LENGTH",12X,"WZ",14X,"WZP",13X,"WZPP"/)
  509 FORMAT(I10,4E16.6)
  500 FORMAT(//,2X,"DELTA=",E12.4,2X,"AL1=",E12.4,2X,"GA1=",E12.4,2X,
     1"AL2=",E12.4,2X,"BT2=",E12.4,2X,"GA2=",E12.4)
  501 FORMAT(//,2X,"H11=",E12.4,2X,"H12=",E12.4,2X,"H22=",E12.4,2X,
     1"Q11=",E12.4,2X,"Q12=",E12.4,2X,"Q22=",E12.4)
  502 FORMAT(//,2X,"D11=",E12.4,2X,"D12=",E12.4,2X,"D22=",E12.4,2X,
     1"DB2=",E12.4,2X,"DB3=",E12.4,2X,"DB4=",E12.4)
  503 FORMAT(//,2X,"DL1=",E12.4,2X,"DL2=",E12.4,2X,"DL3=",E12.4,2X,
     1"DL4=",E12.4,2X,"DL5=",E12.4)
  504 FORMAT(//,2X,"DA1=",E12.4,2X,"DA2=",E12.4,2X,"DA3=",E12.4,2X,
     1"DA4=",E12.4)
  511 FORMAT(//,2X,"FOR FIXED PRESSURE OF ",E12.4,2X,"THE INITIAL AXIAL
     1LOAD IS ",E12.4,2X,"THE INCREMENT OF AXIAL LOAD IS ",E12.4/
     12X,"AND THE ACCURACY (PERCENT) OF THE AXIAL LOAD IS ",E12.4)
  512 FORMAT(//,2X,"FOR FIXED AXIAL LOAD OF ",E12.4,2X,"THE INITIAL HYDR
     1OSTATIC PRESSURE IS",E12.4/2X,"THE INCREMENT OF THE HYDROSTATIC PR
     2ESSURE IS",E12.4,2X,"AND THE ACCURACY (PERCENT) OF THE HYDROSTATIC
     3 PRESSURE IS",E12.4)
  505 FORMAT(//,2X,"THE CIRCUMFERENTIAL WAVE NUMBER=",I6)
  506 FORMAT(//,2X,"C1=",E12.4,2X,"C2=",E12.4,2X,"C3=",E12.4,2X,
     1"C4=",E12.4,2X,"C5=",E12.4,2X,"C6=",E12.4)
  507 FORMAT(//,2X,"C7=",E12.4,2X,"C8=",E12.4,2X,"C9=",E12.4,2X,
     1"C10=",E12.4,2X,"C11=",E12.4,2X,"C12=",E12.4)
  793 FORMAT(//,2X,"ELAPSED TIME=",E12.4,2X,"SECONDS"/)
  201 FORMAT(//,2X,"INITIAL SOLUTION (FROM LINEAR). FOR N=",I8,2X,"NX="
     1,E12.4,2X,"F=",E12.4/2X,"TIME COPUTATION =",E12.4,2X,"SECONDS"/
     22X,"==============================================================
     3=========================================")
  114 FORMAT(//,2X,"END OF THIS CASE BECAUSE ITER GREATER THAN ",I8)
```

```
112 FORMAT(//,2X,"SOLUTION FOR ITERATION ",I8,2X,"N=",I8,2X,"NX=",
   1E12.4,2X,"P=",E12.4/2X,"TIME COMPUTATION =",E12.4,2X,"SECONDS"/
   22X,"==============================================================
   3========================================")
118 FORMAT(//,2X,"ITER=",I8,2X,"WCH=",E15.5,2X,"FCH=",E15.5/)
991 FORMAT(//,2X,"THE SOLUTION IS NOT CONVERGED  APARENTLY THE INITIAL
   1LOAD SHOULD BE LESS THAN ",E12.4)
241 FORMAT(//,2X,"NONLINEAR SOLUTION FOR NX=",E12.4,2X,"P=",E12.4,2X,
   1"N=",I8,2X,"IS OBTAINED BY ",I8,2X,"ITERATIONS"/2X,"TIME COMPUTATI
   2ON=",E12.4,2X,"SECONDS"/2X,"OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOK
   3OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKO")
242 FORMAT(//,2X,"POTENTIAL ENERGY=",E15.5,2X,"END SHORTENING FOR Y=?.
   1 IS",E15.5,2X,"AVERAGE END SHORTENING=",E15.5)
243 FORMAT(//,2X,"DETEMINANT=",E12.4,2X,"IXPM=",I10)
785 FORMAT(//,2X,"CRITICAL LOAD FOR N=",I8,2X,"IS NX=",E12.4,2X,
   1"P=",E12.4/2X,"POTENTIAL ENERGY=",E15.5,2X,"END SHORTENING FOR Y=)
   2. =",E15.5,2X,"AVERAGE END SHORTENING=",E15.5/2X,"OKOKOKOKOKOKOKOK
   3OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOK")
119 FORMAT(5(I8,E15.5))
545 FORMAT(///2X,"FOR N=",I8,2X,"THE LOAD  ",E12.4,2X,"IS OVER THE LIM
   1IT POINT")
573 FORMAT(//,2X,"XLAMD=",E12.4,2X,"YLAMD=",E12.4,2X,"EX=",E12.4,2X,
   1"EY=",E12.4,2X,"RHOX=",E12.4,2X,"RHOY=",E12.4)
582 FORMAT(//,2X,"FOR N=",I8,2X,"NX=",E12.4,2X,"P=",E12.4,2X,"THE SOLUT
   1ION IS NOT CONVERGED"/2X,"PROBABLY THE CRITICAL LOAD IS SMALLER TH
   2AN THESE LOADS")
789 FORMAT(//,2X,"FOR   =",E12.4,2X,"THE MINIMUN POTENTIAL ENERGY IS "
   1,E15.5,2X,"AND THE CRITICAL WAVE NUMBER IS ",I8)
584 FORMAT(//,2X,"DETERMINATION OF THE CRITICAL CIRCUMFERENTIAL WAVE
   1 NUMBER (N)"///2X,"**********************************************
   2*********"/2X,"***********************************************
   3*****************")
513 FORMAT(//,2X,"THE RELATION BETWEEN AXIAL LOAD AND THE PRESSURE IS
   1 NX=",E12.4,2X,"MULTIPLY BY THE PRESSURE",E12.4/2X,
   2"THE INCREMENT OF THE PRESSURE IS",E12.4/2X,
   3"AND THE ACCURACY (PERCENT) OF THE PRESSURE IS",E12.4)
581 FORMAT(//,2X,"WAVE NO =",I10,2X,"LOAD=",E15.5,2X,
   1"POTENTIAL ENERGY=",E15.5,2X,"ELAPSED TIME=",E12.4,2X,"SECONDS")
722 FORMAT(//,2X,"END OF THIS CASE BECAUSE  NUMBER OF LOAD POINTS IS
   1GREATER THAN",I8)
    END
```

8

```
      SUBROUTINE IMPERF
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FACT3/DL5,XL,XH
      COMMON/FILFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/PRES2/WZ(201,5),WZP(201,5),WZPP(200,5)
      DV=1.
      PI=4.*ATAN(DV)
      AC=1.0*XH
      A1=PI*7./4.
      XX=0.
      DO 10 I1=1,NEQPOT
      WZ(I1,2)=-AC*SIN(A1*XX)
      WZP(I1,2)=-AC*A1*COS(A1*XX)
      WZPP(I1,2)=AC*A1*A1*SIN(A1*XX)
      WZ(I1,1)=0.
      WZP(I1,1)=0.
      WZPP(I1,1)=0.
      IF(K1.LE.2)GO TO 50
      DO 11 J1=3,K1
      WZ(I1,J1)=0.
      WZP(I1,J1)=0.
      WZPP(I1,J1)=0.
   11 CONTINUE
   50 CONTINUE
      XX=XX+DELTA
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRANSF(WF,XWF,FF,XFF,NW,NF,NRF,T1,MAXN,IDER,IPRR)
      COMMON/PRES1/WM(200,5),ETM(200,5),WMP(200,5)
      COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/CDISK/I21(501),I22(501)
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FOURIR/KFCUR,K6,K4,K3,K2,K1
      DIMENSION WF(NRF,NW),XWF(NRF,NW),FF(NRF,NF),XFF(NRF,NF),T1(MAXN)
      IF(IPRR.EQ.3) GO TO 278
      DO 10 I1=1,NEQPOT
      NL=MI(I1)
      CALL READMS(22,T1,NL,I1)
      IF(I1.NE.1) GO TO 175
      DO 11 J1=1,K1
      WF(1,J1)=T1(J1)
      WM(I1,J1)=T1(J1+K6)
      XWF(1,J1)=T1(J1+K3-1)
      ETM(I1,J1)=T1(J1+K3+K6-1)
   11 CONTINUE
      DO 12 J1=1,K2
      FF(1,J1)=T1(J1+K1)
      FM(I1,J1)=T1(J1+K1+K6)
      XFF(1,J1)=T1(J1+K4)
      XFM(I1,J1)=T1(J1+K4+K6)
   12 CONTINUE
      GO TO 10
  175 DO 13 J1=1,K1
      WM(I1,J1)=T1(J1)
      ETM(I1,J1)=T1(J1+K3-1)
   13 CONTINUE
      DO 14 J1=1,K2
      FM(I1,J1)=T1(J1+K1)
      XFM(I1,J1)=T1(J1+K4)
   14 CONTINUE
      IF(I1.NE.NEQPOT) GO TO 10
       DO 15 J1=1,K1
      WF(2,J1)=T1(J1+K6)
      XWF(2,J1)=T1(J1+K3+K6-1)
   15 CONTINUE
      DO 16 J1=1,K2
      FF(2,J1)=T1(J1+K1+K6)
      XFF(2,J1)=T1(J1+K4+K6)
   16 CONTINUE
   10 CONTINUE
      IF(IDER.NE.1) GO TO 275
      NEQP=NEQPOT-1
      DO 20 I1=2,NEQP
      DO 21 J1=1,K1
      WMP(I1,J1)=AL1*WM(I1-1,J1)+GA1*WM(I1+1,J1)
   21 CONTINUE
      DO 22 J1=1,K2
      FMP(I1,J1)=AL1*FM(I1-1,J1)+GA1*FM(I1+1,J1)
   22 CONTINUE
   20 CONTINUE
      DO 23 J1=1,K1
      WMP(1,J1)=AL1*WF(1,J1)+GA1*WM(2,J1)
      WMP(NEQPOT,J1)=AL1*WM(NEQP,J1)+GA1*WF(2,J1)
```

```
 23 CONTINUE
    DO 24 J1=1,K2
    FMP(1,J1)=AL1*FF(1,J1)+GA1*FM(2,J1)
    FMP(NEQPOT,J1)=AL1*FM(NEQP,J1)+GA1*FF(2,J1)
 24 CONTINUE
275 IF(IPRR.NE.1)RETURN
278 CONTINUE
    J1=0
    WRITE(6,400)J1
    WRITE(6,500)
    XX=0.
    WRITE(6,600)WF(1,1),XWF(1,1)
    DO 48 I1=1,NEQPOT
    WRITE(6,509)I1,XX,WM(I1,1),WMP(I1,1),ETM(I1,1)
    XX=XX+DELTA
 48 CONTINUE
    WRITE(6,600)WF(2,1),XWF(2,1)
    DO 49 J1=1,KFOUR
    WRITE(6,400)J1
    WRITE(6,500)
    WRITE(6,700)WF(1,J1+1),XWF(1,J1+1),FF(1,J1),XFF(1,J1)
    DO 51 I1=1,NEQPOT
    WRITE(6,609)I1,WM(I1,J1+1),WMP(I1,J1+1),ETM(I1,J1+1),FM(I1,J1),
   1FMP(I1,J1),XFM(I1,J1)
 51 CONTINUE
    WRITE(6,700)WF(2,J1+1),XWF(2,J1+1),FF(2,J1),XFF(2,J1)
 49 CONTINUE
    DO 52 J1=K1,K2
    WRITE(6,400)J1
    WRITE(6,500)
    WRITE(6,800)FF(1,J1),XFF(1,J1)
    DO 53 I1=1,NEQPOT
    WRITE(6,709)I1,FM(I1,J1),FMP(I1,J1),XFM(I1,J1)
 53 CONTINUE
    WRITE(6,800)FF(2,J1),XFF(2,J1)
 52 CONTINUE
400 FORMAT(//,2X,"INTERMIDIAT RESULTS FOR KFOUR=",I8/2X,"*************
   1*******************************")
500 FORMAT(//,2X,"POINT",4X,"LENGTH",10X,"W",14X,"WP",13X,"WPP",
   112X,"F",14X,"FF",13X,"FPP"/2X,"***********************************
   2*******************************************************************
   3********")
509 FORMAT(I8,E12.4,3E15.6)
600 FORMAT(//20H  FICTIVE POINT       E15.6,15X,E15.6//)
609 FORMAT(I8,12X,6E15.6)
700 FORMAT(//20H  FICTIVE POINT       E15.6,15X,2E15.6,15X,E15.6//)
800 FORMAT(//65H  FICTIVE POINT
   1                     E15.6,15X,E15.6//)
709 FORMAT(I8,57X,3E15.6)
    RETURN
    END
```

```
      SUBROUTINE ABCG(IEQ,M1,CF,BF,AF,GF,NRHS,XNXX,LN,NJ,NW,NF)
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/BCUND/LS1,LSN
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/FOURIF/KFCUF,K6,K4,K3,K2,K1
      DIMENSION AF(M1,M1),BF(M1,M1),CF(M1,M1),GF(M1,NRHS)
C    NEQPOT=NPOINT (EXCLUDING FICTIVES POINTS)
C LS1   -KIND OF BOUNDARY CONDITIONS OF POINT 1
C  LSN   -KIND OF BOUNDARY CONDITIONS OF POINT NP
C   NW  -  MAXIMUM  K+1  FOR DIMENSION  WM(NJ,NW)
C  NF   -  MAXIMUM 2*K  FOR DIMENSION  FM(NJ,NF)
      IF(IEQ.GT.1) GO TO 10
      CALL RSTG(BF,CF,AF,GF,1,XNXX,M1,NJ,NW,NF,LN,NRHS)
      DO 2 I1=1,K6
      GF(I1+K6,NRHS)=GF(I1,NRHS)
      DO 2 J1=1,K6
      BF(I1+K6,J1)=AL2*BF(I1,J1)+AL1*CF(I1,J1)
      BF(I1+K6,J1+K6)=BT2*BF(I1,J1)+AF(I1,J1)
      BF(I1,J1+K6)=GA2*BF(I1,J1)+GA1*CF(I1,J1)
    2 CONTINUE
      CALL BCUNDR(AF,CF,GF,1,XNXX,LS1,M1,NJ,NW,NF,LN,NRHS)
      DO 3 I1=1,K6
      DO 3 J1=1,K6
      BF(I1,J1)=AL1*AF(I1,J1)
      AF(I1+K6,J1)=BF(I1,J1+K6)
      BF(I1,J1+K6)=CF(I1,J1)
      AF(I1,J1)=GA1*AF(I1,J1)
    3 CONTINUE
      RETURN
   10 IF(IEQ.GT.2) GO TO 20
      CALL RSTG(AF,CF,BF,GF,2,XNXX,M1,NJ,NW,NF,LN,NRHS)
      DO 4 I1=1,K6
      DO 4 J1=1,K6
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      CF(I1,J1+K6)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      CF(I1,J1)=0.
    4 CONTINUE
      RETURN
   20 IF(IEQ.GE.NEQPOT-1) GO TO 30
      JP=IEQ
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,M1,NJ,NW,NF,LN,NRHS)
      DO 5 I1=1,K6
      DO 5 J1=1,K6
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      TEMP=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      CF(I1,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=TEMP
    5 CONTINUE
      RETURN
   30 IF(IEQ.EQ.NEQPOT) GO TO 40
      JP=IEQ
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,M1,NJ,NW,NF,LN,NRHS)
      DO 6 I1=1,K6
      DO 6 J1=1,K6
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      TEMP=GA2*AF(I1,J1)+GA1*CF(I1,J1)
```
12

13

```
      CF(I1,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=TEMP
      AF(I1,J1+K6)=0.
    6 CONTINUE
      RETURN
   40 JP=IEG
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,M1,NJ,NW,NF,LN,NRHS)
      DO 7 I1=1,K6
      GF(I1+K6,NRHS)=GF(I1,NRHS)
      DO 7 J1=1,K6
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      BF(I1,J1+K6)=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      BF(I1+K6,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
    7 CONTINUE
      CALL BCUNDR(CF,AF,GF,JF,XNXX,LSN,M1,NJ,NW,NF,LN,NRHS)
      DO 8 I1=1,K6
      TEMP=GF(I1,NRHS)
      GF(I1,NRHS)=GF(I1+K6,NRHS)
      GF(I1+K6,NRHS)=TEMP
      DO 8 J1=1,K6
      BF(I1+K6,J1+K6)=GA1*CF(I1,J1)
      CF(I1+K6,J1)=AL1*CF(I1,J1)
      CF(I1,J1)=BF(I1+K6,J1)
      BF(I1+K6,J1)=AF(I1,J1)
    8 CONTINUE
      RETURN
      END
```

```
      FUNCTION ALB(I,J,L,B,JP,N2,N3,N4,LL)
C      N4=1 FOR   B(JP,I+J) OR B(JP,I-J)
C   N4=2  FOR B(JP,J)
C    LL=1 FOR W     LL=2  FOR F
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      DIMENSION B(N2,N3)
      IF(L.GT.3) GO TO 10
      I1=I+J
      I2=I1
      GO TO 20
   10 I1=IABS(I-J)
      I2=I1
   20 IF(N4.EQ.1) GO TO 120
      I2=J
      GO TO 100
  120 IF(I1.LE.KFOUR) GO TO 100
      ALB=0.
      RETURN
  100 IF(L.LE.3) GO TO 110
      ETA=1.
      IF(I.EQ.J) ETA=0.
  110 GO TO(11,12,13,14,15,16),L
  11  R1=I1**2
      GO TO 17
  12  R1=J**2
      GO TO 17
   13 R1=2.*I1*J
      GO TO 17
   14 R1=(2.-ETA)*I1**2
      GO TO 17
   15 R1=(2.-ETA)*J**2
      GO TO 17
   16 IF(I-J.LT.0) ETA=-1.
      R1=-2.*ETA*J*I1
   17 IF(LL.EQ.1)I2=I2+1
      ALB=R1*B(JP,I2)
      RETURN
      END
```

```
      SUBROUTINE RSTG(R,S,T,G,JP,XNXX,M1,NJ,NW,NF,LN,NRHS)
C     LN=1    FOR LINEAR        LN=2  FOR NONLINEAR
C     NJ       MAXIMUM  POINTS IN MERIDIONAL DIRECTION
C   NW     MAXIMUM  KFOUR+1        NF-  MAXIMUM 2.*KFOUR
C    XNXX   AXIAL COMPRESSION LOAD    JP-  THE POINT  IN MERIDIONAL DIRECTION
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/PRES1/WM(200,5),ETM(200,5),WMP(200,5)
      COMMON/PRES2/WZ(200,5),WZP(200,5),WZPP(200,5)
      COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)
      COMMON/XXLOAD/XPRES
      DIMENSION R(M1,M1),S(M1,M1),T(M1,M1),G(M1,NRHS)
C     K6=6*KFOUR+2
C    K4=4*KFOUR+2
C    K3=3*KFOUR+2
C    K2=2*KFOUR
C    K1=KFOUR+1
C    C9=(NNN/RR)**2
C     C1=DD*H11
C    C2=2.*DD*H12*C9
C    C3=2.*Q12*C9
C    C4=DD*H22*C9**2
C     C5=Q11/D11*C9
C    C6=1./(RR*D11)*C9
C    C7=C9**2/(2.*D11)
C     C8=Q22*C9**2
C    C10=C9/2.
C    C11=2.*C9*D12
C    C12=D22*C9**2
      DO 1 I1=1,K6
      G(I1,NRHS)=0.
      DO 1 J1=1,K6
      R(I1,J1)=0.
      S(I1,J1)=0.
      T(I1,J1)=0.
    1 CONTINUE
      J1=0
      DO 2 I1=K3,K6
       T(I1,I1)=1.
      J1=J1+1
      R(I1,J1)=-1.
    2 CONTINUE
C   EQUILIBRIUM EQUATION   FOR I=0
      R(1,K3)=DD*H11+Q11**2/D11
      T(1,K3)=2.*Q11/(RR*D11)+XNXX
      T(1,1)=1./(RR**2*D11)
      G(1,NRHS)=-XNXX*WZPP(JP,1)
      G(1,NRHS)=G(1,NRHS)+XPRES
      DO 6 J=1,KFOUR
      IS=J**2
      M=J+1
      T(1,K3+J)=T(1,K3+J)-C5/2.*IS*WZ(JP,M)
      S(1,J+1)=S(1,J+1)-C5*IS*WZP(JP,M)
      T(1,J+1)=T(1,J+1)-IS/2.*(C5*WZPP(JP,M)+C6*WZ(JP,M))
      T(1,K4+J)=T(1,K4+J)+C10*IS*WZ(JP,M)
      T(1,K1+J)=T(1,K1+J)+C10*IS*WZPP(JP,M)
```

AD-A039 148 GEORGIA INST OF TECH ATLANTA SCHOOL OF ENGINEERING S--ETC F/G 20/11
THE EFFECT OF INITIAL IMPERFECTIONS ON OPTIMAL STIFFENED CYLIND--ETC(U)
FEB 77 G J SIMITSES, I SHEINMAN AF-AFOSR-2655-74

UNCLASSIFIED AFOSR-TR-77-0639 NL

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      S(1,K1+J)=S(1,K1+J)+2.*C10*IS*WZP(JP,M)
      IF(LN.EQ.1) GO TO 6
      T(1,K3+J)=T(1,K3+J)-C5/2.*IS*WM(JP,M)+C10*IS*FM(JP,J)
      T(1,J+1)=T(1,J+1)-IS/2.*(C5*ETM(JP,M)+C6*WM(JP,M))
     1 +C10*IS*XFM(JP,J)
      S(1,J+1)=S(1,J+1)-C5*IS*WMP(JP,M)+2.*C10*IS*FMP(JP,J)
      T(1,K4+J)=T(1,K4+J)+C10*IS*WM(JP,M)
      T(1,K1+J)=T(1,K1+J)+C10*IS*ETM(JP,M)
      S(1,K1+J)=S(1,K1+J)+2.*C10*IS*WMP(JP,M)
      G(1,NRHS)=G(1,NRHS)-C5/2.*IS*(WM(JP,M)*ETM(JP,M)+WMP(JP,M)**2)
     1-C6/4.*IS*(WM(JP,M)**2)+C10*IS*(WM(JP,M)*XFM(JP,J)+ETM(JP,M)*
     2FM(JP,J)+2.*WMP(JP,M)*FMP(JP,J))
    6 CONTINUE
C   EQUILIBRIUM EQUATIONS FOR I=1,2,,,,,,,KFOUR
      DO 3 I1=2,K1
      I=I1-1
      IS=I**2
      IS2=IS**2
      T(I1,1)=-C6*IS*WZ(JP,I1)
      T(I1,K3)=-C5*IS*WZ(JP,I1)
      T(I1,I1)=C4*IS2
      T(I1,I1+K1-1)=-C8*IS2
      T(I1,I1+K3-1)=-C2*IS+XNXX
      T(I1,I1+K4-1)=C3*IS-1./RR
      R(I1,I1+K3-1)=C1
      R(I1,I1+K4-1)=-Q11
      E1=C7*IS*WZ(JP,I1)
      G(I1,NRHS)=-XNXX*WZPP(JP,I1)
      IF(LN.EQ.1) GO TO 60
      T(I1,1)=T(I1,1)-C6*IS*WM(JP,I1)
      T(I1,K3)=T(I1,K3)-C5*IS*WM(JP,I1)
      T(I1,I1)=T(I1,I1)-IS*(C5*ETM(JP,1)+C6*WM(JP,1))
      E1=C7*IS*(WM(JP,I1)+WZ(JP,I1))
      E2=C7/2.*IS
      E3=E2*WM(JP,I1)
      G(I1,NRHS)=G(I1,NRHS)-C5*IS*ETM(JP,1)*WM(JP,I1)
     1 -C6*IS*WM(JP,1)*WM(JP,I1)
   60 DO 4 J=1,KFOUR
      JS=J**2
       M=J+1
      T(I1,J+1)=T(I1,J+1)+E1*JS*WZ(JP,M)
      IF(LN.EQ.1) GO TO 4
      T(I1,J+1)=T(I1,J+1)+E1*JS*WM(JP,M)
      T(I1,I1)=T(I1,I1)+E2*JS*WM(JP,M)*(WM(JP,M)+2.*WZ(JP,M))
      G(I1,NRHS)=G(I1,NRHS)+E1/2.*JS*WM(JP,M)**2+E3*JS*WM(JP,M)
     1*(WM(JP,M)+2.*WZ(JP,M))
    4 CONTINUE
      DO 5 J=1,K2
      T(I1,K4+J)=T(I1,K4+J)+C10*(ALB(I,J,1,WZ,JP,NJ,NW,1,1)+
     1ALB(I,J,4,WZ,JP,NJ,NW,1,1))
      T(I1,K1+J)=T(I1,K1+J)+C10*(ALB(I,J,2,WZPP,JP,NJ,NW,1,1)+
     1 ALB(I,J,5,WZPP,JP,NJ,NW,1,1))
      S(I1,K1+J)=S(I1,K1+J)+C10*(ALB(I,J,3,WZP,JP,NJ,NW,1,1)+
     1 ALB(I,J,6,WZP,JP,NJ,NW,1,1))
      IF(LN.EQ.1) GO TO 5
      T(I1,K4+J)=T(I1,K4+J)+C10*(ALB(I,J,1,WM,JP,NJ,NW,1,1)+
     1 ALB(I,J,4,WM,JP,NJ,NW,1,1))
```

```
      T(I1,K1+J)=T(I1,K1+J)+C10*(ALB(I,J,2,ETM,JP,NJ,NW,1,1)+
     1 ALB(I,J,5,ETM,JP,NJ,NW,1,1))
      S(I1,K1+J)=S(I1,K1+J)+C10*(ALB(I,J,3,WMP,JP,NJ,NW,1,1)+
     1 ALB(I,J,6,WMP,JP,NJ,NW,1,1))
      G(I1,NRHS)=G(I1,NRHS)+C10*((ALB(I,J,1,WM,JP,NJ,NW,1,1)+
     1 ALB(I,J,4,WM,JP,NJ,NW,1,1))*XFM(JP,J)+(ALB(I,J,2,ETM,JP,NJ,NW,
     21,1)+ALB(I,J,5,ETM,JP,NJ,NW,1,1))*FM(JP,J)+(ALB(I,J,3,WMP,JP,NJ,
     3 NW,1,1)+ALB(I,J,6,WMP,JP,NJ,NW,1,1))*FMP(JP,J))
      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 80
      T(I1,IJ1+1)=T(I1,IJ1+1)+C10*(ALB(I,J,1,XFM,JP,NJ,NF,2,2))
      T(I1,K3+IJ1)=T(I1,K3+IJ1)+C10*ALB(I,J,2,FM,JP,NJ,NF,2,2)
      S(I1,IJ1+1)=S(I1,IJ1+1)+C10*ALB(I,J,3,FMP,JP,NJ,NF,2,2)
   80 IJ2=IABS(I-J)
      IF(IJ2.GT.KFOUR) GO TO 5
      T(I1,IJ2+1)=T(I1,IJ2+1)+C10*ALB(I,J,4,XFM,JP,NJ,NF,2,2)
      T(I1,K3+IJ2)=T(I1,K3+IJ2)+C10*ALB(I,J,5,FM,JP,NJ,NF,2,2)
      S(I1,IJ2+1)=S(I1,IJ2+1)+C10*ALB(I,J,6,FMP,JP,NJ,NF,2,2)
    5 CONTINUE
    3 CONTINUE
C     COMPATIBILITY EQUATIONS FOR I=1,2,,,,,,2KFOUR
      E1=C10/2.
      I2=K1+1
      I3=I2+K2-1
      DO 7 I1=I2,I3
      I=I1-K1
      IS=I**2
      R(I1,I1+K4-K1)=D11
      T(I1,I1+K4-K1)=-IS*C11
      T(I1,I1)=IS**2*C12
      IF(I.GT.KFOUR) GO TO 82
      R(I1,I1+K3-K1)=Q11
      T(I1,I1+K3-K1)=-IS*C3+1./RR
      T(I1,I1-K1+1)=IS**2*C8
   82 DO 9 J1=1,K1
      J=J1-1
      T(I1,J1+K3-1)=T(I1,J1+K3-1)-C10*(ALB(I,J,1,WZ,JP,NJ,NW,1,1)+
     1ALB(I,J,4,WZ,JP,NJ,NW,1,1))
      T(I1,J1)=T(I1,J1)-C10*(ALB(I,J,2,WZPP,JP,NJ,NW,1,1)+
     1 ALB(I,J,5,WZPP,JP,NJ,NW,1,1))
      S(I1,J1)=S(I1,J1)-C10*(ALB(I,J,3,WZP,JP,NJ,NW,1,1)+
     1 ALB(I,J,6,WZP,JP,NJ,NW,1,1))
      IF(LN.EQ.1) GO TO 9
      T(I1,J1+K3-1)=T(I1,J1+K3-1)-E1*(ALB(I,J,1,WM,JP,NJ,NW,1,1)+
     1 ALB(I,J,4,WM,JP,NJ,NW,1,1))
      T(I1,J1)=T(I1,J1)-E1*(ALB(I,J,2,ETM,JP,NJ,NW,1,1)+
     1 ALB(I,J,5,ETM,JP,NJ,NW,1,1))
      S(I1,J1)=S(I1,J1)-E1*(ALB(I,J,3,WMP,JP,NJ,NW,1,1)+
     1 ALB(I,J,6,WMP,JP,NJ,NW,1,1))
      G(I1,NRHS)=G(I1,NRHS)-E1*((ALB(I,J,1,WM,JP,NJ,NW,1,1)
     1+ALB(I,J,4,WM,JP,NJ,
     2NW,1,1))*ETM(JP,J1)+(ALB(I,J,2,ETM,JP,NJ,NW,1,1) +ALB(I,J,5,ETM,
     3JP,NJ,NW,1,1))*WM(JP,J1)+(ALB(I,J,3,WMP,JP,NJ,NW,1,1)+ALB(I,J,6,
     4WMP,JP,NJ,NW,1,1))*WMP(JP,J1))
      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 83
      T(I1,IJ1+1)=T(I1,IJ1+1)-E1*(ALB(I,J,1,ETM,JP,NJ,NW,2,1))
```

```
      T(I1,K3+IJ1)=T(I1,K3+IJ1)-E1*(ALB(I,J,2,WM,JP,NJ,NW,2,1))
      S(I1,IJ1+1)=S(I1,IJ1+1)-E1*ALB(I,J,3,WMP,JP,NJ,NW,2,1)
   83 IJ2=IABS(I-J)
      IF(IJ2.GT.KFCUR) GO TO 9
      T(I1,IJ2+1)=T(I1,IJ2+1)-E1*ALB(I,J,4,ETM,JP,NJ,NW,2,1)
      T(I1,K3+IJ2)=T(I1,K3+IJ2)-E1*ALB(I,J,5,WM,JP,NJ,NW,2,1)
      S(I1,IJ2+1)=S(I1,IJ2+1)-E1*ALB(I,J,6,WMP,JP,NJ,NW,2,1)
    9 CONTINUE
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BCUNDR(BS,BT,BG,IN,XNXX,LS,M1,NJ,NW,NF,LN,NRHS)
      COMMCN/FOURIR/KFCUR,K6,K4,K3,K2,K1
      COMMCN/PRES1/WM(200,5),ETM(200,5),WMP(2.0,5)
      COMMCN/PRES2/WZ(200,5),WZP(200,5),WZPP(200,5)
      COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)
      COMMON/GECM/RR,DC,H11,H12,H22,Q11,Q12,C22,D11,D12,D22
      COMMCN/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/FACTCR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      DIMENSION BS(M1,M1),BT(M1,M1),BG(M1,NRHS)
C     BCUNDARY CCNDITIONS   BS*Z;+BT*Z=BG
C    LS=1 FOR  SS1   W=MX=NXY=NX=0.
C  LS=2 FOR SS2   W=MX=NXY=U=0.
C    LS=3 FCR SS3   W=MX=V=NX=0.
C    LS=4  FCR SS4   W=MX=V=U=0.
C    LS=5 FOR CC1   W=W,X=NXY=NX=0.
C    LS=6  FCR CC2  W=W,X=NXY=U=0.
C     LS=7  FCR CC3    W=W,X=V=NX=0.
C     LS=8  FCR CC4    W=W,X=V=U=0.
C    LS=9 FOR FREE EDGE  NX=NXY=QX=MX=0.
C    LS=10 FOR SYMMETRY   NXY=QX=W,X=U=0.
C    LS=11 FCR ANTISYMMETRY  NX=MX=W=V=0.
C    DALFA=(1.+XLAMD)*(1.+YLAMD)-XNI**2
C    DL1=DD*(1.+RHOX+EX**2*XLAMD*(1.+YLAMD-XNI**2)*12./(XH**2*DALFA))
C    DL2=DD*XNI*(1.+EX*EY*YLAMD*XLAMD*12./(XH**2*DALFA))
C    DL3=EX*XLAMD*(1.+YLAMD)/DALFA
C    DL4=-XNI*EX*XLAMD/DALFA
C  DA1=(1.+YLAMD)/(DALFA*EXXP)
C    DA2=-XNI/(DALFA*EXXP)
C    DA3=-(1.+YLAMD)*EX*XLAMD/DALFA
C    DA4=XNI*EY*YLAMD/DALFA
C  DB2=(1.+XLAMD)/(DALFA*EXXP)
C    DB3=XNI*EX*XLAMD/DALFA
C    DB4=-(1.+XLAMD)*EY*YLAMD/DALFA
C      K6=6*KFOUR+2
C    K4=4*KFCUR+2
C    K3=3*KFCUR+2
C    K2=2*KFCUR
C    K1=KFOUR+1
      IF(LS.EQ.11)LS=3
      DO 1 I1=1,K6
      BG(I1,NRHS)=0.
      DO 1 J1=1,K6
      BS(I1,J1)=0.
      BT(I1,J1)=0.
    1 CONTINUE
      IF(LS.EQ.10) GO TO 888
      IF(LS.GT.8) GO TC 100
      DO 2 I1=1,K1
    2 BT(I1,I1)=1.
      IF(LS.LE.4.OR.LS.GT.8) GO TO 100
  888 J=0
      DO 3 I1=K3,K4
      J=J+1
    3 BS(I1,J)=1.
  100 IF(LS.GT.4) GO TC 200
      BT(K3,K3)=1.
      K31=K3+1
```

```
      DO 4 I1=K31,K4
      BT(I1,I1)=DL1
      BT(I1,I1+K4-K3)=DL4
      IF(LS.EQ.1.OR.LS.EQ.3) GO TO 4
      I=I1-K3
      BT(I1,I1+K1-K3)=BT(I1,I1+K1-K3)-I**2*C9*DL3
    4 CONTINUE
  200 IF(LS.NE.10) GO TO 300
      E1=DL4/D11*C10
      BS(1,K3)=DL1-DL4/D11*D11
      DO 5 J1=1,KFOUR
      JS=J1**2
      BS(1,J1+1)=BS(1,J1+1)+E1*JS*WZ(IN,J1+1)
      BT(1,J1+1)=BT(1,J1+1)+E1*JS*WZP(IN,J1+1)
      BT(1,K1+J1)=BT(1,K1+J1)+C10*JS*WZP(IN,J1+1)
      IF(LN.EQ.1) GO TO 5
      BS(1,J1+1)=BS(1,J1+1)+E1*JS*WM(IN,J1+1)
      BT(1,J1+1)=BT(1,J1+1)+E1*JS*WMP(IN,J1+1)
      BG(1,NRHS)=BG(1,NRHS)+E1*JS*WM(IN,J1+1)*WMP(IN,J1+1)
    5 CONTINUE
      DO 12 I1=2,K1
      I=I1-1
      BS(I1,I1+K3-1)=BS(I1,I1+K3-1)+DL1
      BS(I1,I1+K4-1)=BS(I1,I1+K4-1)+DL4
      DO 12 J1=1,K2
      BT(I1,K1+J1)=BT(I1,K1+J1)+C10*(ALB(I,J1,2,WZP,IN,NJ,NW,1,1)+
     1 ALB(I,J1,5,WZP,IN,NJ,NW,1,1))
   12 CONTINUE
  300 IF(LS.NE.9) GO TO 400
      E1=DL4/D11*D11
      BT(1,K3)=DL1-E1
      BS(K3,K3)=DL1-E1
      E1=DL4/(D11*RR)
      BT(1,1)=-E1
      BS(K3,1)=XNXX-E1
      BG(K3,NRHS)=-XNXX*WZP(IN,1)
      E1=DL4/D11*C10
      DO 6 J1=1,KFOUR
      JS=J1**2
      BT(1,J1+1)=BT(1,J1+1)+E1*JS*WZ(IN,J1+1)
      BT(K3,J1+1)=BT(K3,J1+1)+E1*JS*WZP(IN,J1+1)
      BS(K3,J1+1)=BS(K3,J1+1)+E1*JS*WZ(IN,J1+1)
      IF(LN.EQ.1) GO TO 6
      BT(1,J1+1)=BT(1,J1+1)+E1*JS*WM(IN,J1+1)
      BG(1,NRHS)=BG(1,NRHS)+E1/2.*JS*WM(IN,J1+1)**2
      BT(K3,J1+1)=BT(K3,J1+1)+E1*JS*WMP(IN,J1+1)
      BS(K3,J1+1)=BS(K3,J1+1)+E1*JS*WM(IN,J1+1)
      BG(K3,NRHS)=BG(K3,NRHS)+E1*JS*WM(IN,J1+1)*WMP(IN,J1+1)
    6 CONTINUE
      DO 13 I1=2,K1
      I=I1-1
      IS=I**2
      BT(I1,I1+K3-1)=DL1
      BT(I1,I1)=-IS*C9*DL2
      BT(I1,I1+K4-1)=DL4
      BS(I1+K3-1,I1+K4-1)=DL4
      BS(I1+K3-1,I1+K3-1)=DL1
```

```
      BS(I1+K3-1,I1)=-IS*C9*(DL2+2.*DD*(1.-XNI))+XNXX
      BG(I1+K3-1,NRHS)=-XNXX*WZP(IN,I1)
   13 CONTINUE
  400 I2=K1+1
      I3=I2+K2-1
      I4=K3-1
      DO 7 I1=I2,I3
      I=I1-K1
      IS=I**2
      GO TO (21,22,23,24,21,26,27,28,21,30),LS
   21 BS(I1,K1+I)=1.
      BT(I1+I4,K1+I)=1.
      GO TO 7
   22 BS(I1,K1+I)=1.
      BS(I1+I4,K4+I)=DB2
      DO 8 J1=1,K1
      J=J1-1
      BS(I1+I4,J1)=BS(I1+I4,J1)-C10*(ALB(I,J,1,WZ,IN,NJ,NW,1,1)+
     1ALB(I,J,4,WZ,IN,NJ,NW,1,1))
    8 CONTINUE
      IF(I.GT.KFOUR) GO TO 7
      BS(I1+I4,K3+I)=BS(I1+I4,K3+I)+DB3
      BS(I1+I4,I+1)=BS(I1+I4,I+1)-IS*C9*DB4+1./RR
      GO TO 7
   23 BT(I1,K1+I)=1.
      BT(I1+I4,K4+I)=DB2
      IF(I.GT.KFOUR) GO TO 7
      BT(I1+I4,K3+I)=DB3
      GO TO 7
   24 BT(I1,K1+I)=-IS*C9*DA2
      BT(I1,K4+I)=DB2
      BS(I1+I4,K1+I)=-IS*C9*(DA2+2./((1.-XNI)*EXXP))
      BS(I1+I4,K4+I)=DB2
      DO 9 J1=1,K1
      J=J1-1
      BS(I1+I4,J1)=BS(I1+I4,J1)-C10*(ALB(I,J,1,WZ,IN,NJ,NW,1,1)+
     1 ALB(I,J,4,WZ,IN,NJ,NW,1,1))
    9 CONTINUE
      IF(I.GT.KFOUR) GO TO 7
      BT(I1,K3+I)=DB3
      BS(I1+I4,K3+I)=DB3
      BS(I1+I4,I+1)=BS(I1+I4,I+1)-IS*C9*DB4+1./RR
      GO TO 7
   26 BS(I1,K1+I)=1.
      BS(I1+I4,K4+I)=DB2
      IF(I.GT.KFOUR) GO TO 7
      BS(I1+I4,K3+I)=DB3
      GO TO 7
   27 BT(I1,K1+I)=1.
      BT(I1+I4,K4+I)=DB2
      IF(I.GT.KFOUR) GO TO 7
      BT(I1+I4,K3+I)=DB3
      GO TO 7
   28 BT(I1,K1+I)=-IS*C9*DA2
      BT(I1,K4+I)=DB2
      BS(I1+I4,K1+I)=-IS*C9*(DA2+2./((1.-XNI)*EXXP))
      BS(I1+I4,K4+I)=DB2
```

```
      IF(I.GT.KFOUR) GO TO 7
      BT(I1,K3+I)=DB3
      BS(I1+I4,K3+I)=DB3
      GO TO 7
   30 BS(I1,K1+I)=1.
      BS(I1+I4,K4+I)=DB2
      DO 14 J1=1,KFOUR
      BT(I1+I4,J1+1)=BT(I1+I4,J1+1)-C10*(ALB(I,J1,2,WZP,IN,NJ,NW,1,1)+
     1ALB(I,J1,5,WZP,IN,NJ,NW,1,1))
   14 CONTINUE
      BS(I1+I4,K3+I)=BS(I1+I4,K3+I)+DB3
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INVERT(NA,A,C,M,NM1,NM2,DET,IXP,IDET)
      DIMENSION A(NM1,NM1),C(NM2),M(NM2)
      DET=1.
      IXP=0
      NN=NA
      IF(NN.NE.1) GO TO 303
      DET=A(1,1)
      A(1,1)=1./A(1,1)
      GO TO 304
  303 DO 90 I=1,NN
   90 M(I)=-I
      DO 140 II=1,NN
      D=0.D0
      DO 112 K=1,NN
      IF(M(K)) 100,100,112
  100 DO 110 L=1,NN
      IF(M(L)) 103,103,110
  103 IF(ABS(D)-ABS(A(K,L)))105,105,110
  105 LD=L
      KD=K
      D=A(K,L)
      BIGA=D
  110 CONTINUE
  112 CONTINUE
      IF(D.EQ.0.D0) GO TO 170
      GO TO 188
  170 WRITE(6,502)
      STOP
  502 FORMAT(/,5X,"DETERMINANT=0"/)
  188 NEMP=-M(LD)
      M(LD)=M(KD)
      M(KD)=NEMP
      DO 114 I=1,NN
      C(I)=A(I,LD)
      A(I,LD)=A(I,KD)
  114 A(I,KD)=0.D0
      A(KD,KD)=1.D0
      DO 115 J=1,NN
  115 A(KD,J)=A(KD,J)/D
      DO 135 I=1,NN
      IF(I.EQ.KD) GO TO 135
      DO 134 J=1,NN
      TEMP=C(I)*A(KD,J)
  134 A(I,J)=A(I,J)-TEMP
  135 CONTINUE
      IF(IDET.NE.1) GO TO 140
      DET=DET*BIGA
      IF(KD.NE.LD)DET=-DET
  629 IF(ABS(DET).LT.1.E+10) GO TO 630
      DET=DET/1.E+10
      IXP=IXP+10
      GO TO 629
  630 IF(ABS(DET).GT.1.E-10) GO TO 140
      DET=DET*1.E+10
      IXP=IXP-10
  140 CONTINUE
      DO 2[illegible] I=1,NN
```

```
      L=0
  150 L=L+1
      IF(M(L)-I) 150,160,150
  160 M(L)=M(I)
      M(I)=I
      DO 200 J=1,NN
      TEMP=A(L,J)
       A(L,J)=A(I,J)
  200 A(I,J)=TEMP
  304 RETURN
      END
```

2H

25

```
      SUBROUTINE YMY(N1,A,B,C,N2,L1,L2,L3,T)
      DIMENSION A(L1,L2),B(L1,L1),C(L1,L2),T(L3)
      IF(N2.EQ.1) GO TO 100
      DO 11 I=1,N1
      DO 10 J=1,N2
      TEMP=0.
      DO 20 K=1,N1
   20 TEMP=TEMP+B(I,K)*C(K,J)
   10 T(J)=TEMP
      DO 30 J=1,N2
   30 A(I,J)=T(J)
   11 CONTINUE
      RETURN
  100 DO 111 I=1,N1
      TEMP=0.
      DO 120 K=1,N1
  120 TEMP=TEMP+B(I,K)*C(K,1)
  111 T(I)=TEMP
      DO 130 I=1,N1
  130 A(I,1)=T(I)
      RETURN
      END
```

```
      SUBROUTINE YSYMY(N2,N1,A,B,C,D,N3,L1,L2,L3,L4,T)
      DIMENSION A(L1,L3),B(L1,L3),C(L1,L2),D(L2,L3),T(L4)
       IF(N3.EQ.1) GO TO 100
      DO 11 I=1,N1
      DO 10 J=1,N3
      TEMP=0.
      DO 20 K=1,N2
   20 TEMP=TEMP+C(I,K)*D(K,J)
   10 T(J)=B(I,J)-TEMP
      DO 30 J=1,N3
   30 A(I,J)=T(J)
   11 CONTINUE
      RETURN
  100 DO 111 I=1,N1
      TEMP=0.
      DO 120 K=1,N2
  120 TEMP=TEMP+C(I,K)*D(K,1)
  111 T(I)=B(I,1)-TEMP
      DO 130 I=1,N1
  130 A(I,1)=T(I)
      RETURN
      END
```

```
      SUBROUTINE POTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,C,MT,T1,
     1 V1,MAX2,IXPM,DETM,XNXX,LN,NJ,NW,NF)
C     MAX2=MAXN*MAXN
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/CDISK/I21(501),I22(501)
      DIMENSION AP(MAXN,MAXN),BP(MAXN,MAXN),CP(MAXN,MAXN)
      DIMENSION PR(MAXN,MAXN),GP(MAXN,NRHS),XP(MAXN,NRHS)
      DIMENSION T1(MAXN),C(MAXN),MT(MAXN),V1(MAX2)
C     EQUIVALENCE (AP(1,1),V1(1))
      IXPM=0
      DETM=1.
      DO 100 I=1,NEQPOT
      CALL AECG(I,MAXN,CP,BP,AP,GP,NRHS,XNXX,LN,NJ,NW,NF)
      N=MI(I)
      IF(I.EQ.1) GO TO 888
      NMIN1=MI(I-1)
  888 IF(I.EQ.NEQPOT) GO TO 999
      NPLUS1=MI(I+1)
  999 CONTINUE
      IF(I.EQ.1) GO TO 12
      CALL YSYMY(NMIN1,N,BP,BP,CP,PR,N,MAXN,MAXN,MAXN,MAXN,T1)
   12 CALL INVERT(N,BP,C,MT,MAXN,MAXN,DET,IXP,IDET)
      IF(IDET.NE.1) GO TO 640
      DETM=DET*DETM
      IXPM=IXP+IXPM
      IF(ABS(DETM).LT.1.E+10) GO TO 630
      DETM=DETM/1.E+10
      IXPM=IXPM+10
      GO TO 640
  630 IF(ABS(DETM).GT.1.E-10) GO TO 640
      DETM=DETM*1.E+10
      IXPM=IXPM-10
  640 CONTINUE
      IF(I.EQ.NEQPOT) GO TO 102
      CALL YMY(N,PR,BP,AP,NPLUS1,MAXN,MAXN,MAXN,T1)
      CALL XWRITE(21,PR,N,NPLUS1,MAXN,MAXN,I,MAX2,V1)
  102 IF(I.EQ.1) GO TO 32
      CALL YSYMY(NMIN1,N,XP,GP,CP,XP,NRHS,MAXN,MAXN,NRHS,MAXN,T1)
      CALL YMY(N,XP,BP,XP,NRHS,MAXN,NRHS,MAXN,T1)
      GO TO 42
   32 CALL YMY(N,XP,BP,GP,NRHS,MAXN,NRHS,MAXN,T1)
   42 CALL XWRITE(22,XP,N,NRHS,MAXN,NRHS,I,MAXN,T1)
  100 CONTINUE
      MEQPOT=NEQPOT-1
      DO 200 K=1,MEQPOT
      NK=NEQPOT-K
      NMIN1=MI(NK)
      N=MI(NK+1)
      CALL XREAD(21,PR,NMIN1,N,MAXN,MAXN,NK,MAX2,V1)
      CALL XREAD(22,GP,NMIN1,NRHS,MAXN,NRHS,NK,MAXN,T1)
      CALL YSYMY(N,NMIN1,XP,GP,PR,XP,NRHS,MAXN,MAXN,NRHS,MAXN,T1)
      CALL XWRITE(22,XP,NMIN1,NRHS,MAXN,NRHS,NK,MAXN,T1)
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XREAD(ND,A,L1,L2,M1,M2,IND,M3,VV)
      COMMON/CDISK/I21(501),I22(501)
      DIMENSION A(M1,M2),VV(M3)
C     RECORD  IND OF DIRECT ACCESS DATA SET ND IS READ AND ALLOCATED
C   BY ROWS INTO L1*L2 PORTION OF MATRIX A
       L3=L1*L2
      CALL READMS(ND,VV,L3,IND)
      KL=0
      DO 10 NROW=1,L1
      DO 10 NCOL=1,L2
      KL=KL+1
      A(NROW,NCOL)=VV(KL)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XWRITE(ND,A,L1,L2,M1,M2,IND,M3,VV)
      COMMON/CDISK/I21(501),I22(501)
      DIMENSION A(M1,M2),VV(M3)
C  L1*L2 PORTION OF MATRIX A IS WRITTEN BY ROWS ON DIRECT ACCESS
C   DATA SET ND IN RECORD IND
      KL=0
      DO 10 NROW=1,L1
      DO 10 NCOL=1,L2
      KL=KL+1
      VV(KL)=A(NROW,NCOL)
   10 CONTINUE
      CALL WRITMS(ND,VV,KL,IND,-1)
      RETURN
      END
```

```
      SUBROUTINE PUTSN(FCT,STRY,STRA,IP,ISY,ISA,XNXX)
C     FCT   - POTENTIAL ENERGY
C     STRY  -  UNIT END SHORTENING FOR Y=0.
C     STRA  -  AVERAGE UNIT END SHORTENING
C     IP=1  FOR CALCULATE POT
C ISY=1    FOR CALCULATE STRY
C     ISA=1 FOR CALCULATE STRA
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/PRES1/WM(200,5),ETM(200,5),WMP(200,5)
      COMMON/PRES2/WZ(200,5),WZP(200,5),WZPP(200,5)
      COMMON/PRES3/FM(200,8),XFM(200,8),FMP(200,8)
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/FACT3/DL5,XL,XH
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/XXLOAD/XPRES
      CE1=C10/2.
      CE2=C9**2/2.
      CE3=C9/((1.-XNI)*EXXP)
      CE4=C9*DD*(1.-XNI)
      POT=0.
      STRY=0.
      STRA=0.
       DO 10 I1=1,NEGPOT
      E7=1.
      IF(I1.EQ.1.OR.I1.EQ.NEGPOT)E7=0.5
      E1=-C11*ETM(I1,1)-1./RR*WM(I1,1)
      E2=0.
      DO 11 J1=1,KFOUR
      JS=J1**2
      E2=JS*WM(I1,J1+1)*(WM(I1,J1+1)+2.*WZ(I1,J1+1))+E2
   11 CONTINUE
      E1=E1+CE1*E2
      IF(IP.NE.1) GO TO 100
      PE1=DB2/D11**2*E1**2+DL1*ETM(I1, 1)**2-XNXX*WMP(I1,1)*(WMP(I1,1)+
     12.*WZP(I1,1))
      PE1=PE1-2.*XPRES*WM(I1,1)
  100 E1=E1*DA2/D11+DA3*ETM(I1,1)
      IF(ISY.NE.1) GO TO 110
      PSY=E1
  110 IF(ISA.NE.1) GO TO 120
      PSA=E1-WMP(I1,1)*(WMP(I1,1)+2.*WZP(I1,1))/2.
  120 IF(ISY.NE.1) GO TO 130
      E1=0.
      DO 18 J1=1,K1
      DO 18 J2=1,K1
      E1=WMP(I1,J1)*(WMP(I1,J2)+2.*WZP(I1,J2))+E1
   18 CONTINUE
      PSY=PSY-E1/2.
      E1=0.
      E2=0.
      DO 12 J1=1,KFOUR
       JS=J1**2
      E1=ETM(I1,J1+1)+E1
      E2=E2+JS*WM(I1,J1+1)
```

```
 12 CONTINUE
    PSY=PSY+DA3*E1-DA4*C9*E2
    E1=0.
    E2=0.
    DO 13 J1=1,K2
    JS=J1**2
    E1=E1+XFM(I1,J1)
    E2=E2+JS*FM(I1,J1)
 13 CONTINUE
    PSY=PSY+DA2*E1-DA1*C9*E2
    STRY=STRY+PSY*E7
130 IF(ISA.NE.1) GO TO 140
    E1=0.
     DO 14 J1=1,KFOUR
 14 E1=E1+WMF(I1,J1+1)*(WMF(I1,J1+1)+2.*WZF(I1,J1+1))
    PSA=PSA-E1/4.
    STRA=STRA+PSA*E7
140 IF(IP.NE.1) GO TO 10
    E1=0.
    E2=0.
     E3=0.
    E4=0.
     E5=0.
    DO 15 J1=1,KFOUR
    JS=J1**2
    JS2=JS**2
    E1=E1+WMP(I1,J1+1)*(WMF(I1,J1+1)+2.*WZF(I1,J1+1))
    E2=E2+JS2*WM(I1,J1+1)**2
    E3=E3+JS*WMP(I1,J1+1)**2
    E4=E4+ETM(I1,J1+1)**2
    E5=E5+JS*WM(I1,J1+1)*ETM(I1,J1+1)
 15 CONTINUE
    PE1=PE1-XNXX*E1/2.+CE2*DL5*E2+CE4*E3+DL1*E4/2.-C9*DL2*E5
    E1=0.
    E2=0.
    E3=0.
    E4=0.
    DO 16 J1=1,K2
    JS=J1**2
    JS2=JS**2
    E1=JS2*FM(I1,J1)**2+E1
    E2=E2+JS*FMP(I1,J1)**2
    E3=E3+XFM(I1,J1)**2
    E4=E4+JS*FM(I1,J1)*XFM(I1,J1)
 16 CONTINUE
    PE1=PE1+CE2*DA1*E1+CE3*E2+DB2*E3/2.-DA2*C9*E4
    POT=POT+PE1*E7
 10 CONTINUE
     IF(IP.NE.1) GO TO 150
    POT=POT*3.14159*RR*DELTA
150 IF(ISY.NE.1) GO TO 160
    STRY=DA1*XNXX-STRY/XL
160 IF(ISA.NE.1) GO TO 170
    STRA=DA1*XNXX-STRA/XL
170 CONTINUE
    RETURN
     END
```

```
      SUBROUTINE CCEFF(EX,EY,XLAMD,YLAMD,RHOX,RHOY,ELAS)
      COMMON/GEOM/RR,DC,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/FOURIR/KFOUR,K6,K4,K3,K2,K1
      COMMON/FACT3/DL5,XL,XH
      K6=6*KFOUR+2
      K4=4*KFOUR+2
      K3=3*KFOUR+2
      K2=2*KFOUR
      K1=KFOUR+1
      XN2=XNI**2
      XH2=XH**2
      DALFA=(1.+XLAMD)*(1.+YLAMD)-XN2
      DD=ELAS*XH**3/(12.*(1.-XN2))
      EXXP=ELAS*XH/(1.-XN2)
      H11=1.+RHOX+12.*EX**2*XLAMD*(1.+YLAMD-XN2)/(XH2*DALFA)
      H22=1.+RHOY+12.*EY**2*YLAMD*(1.+XLAMD-XN2)/(XH2*DALFA)
      H12=1.+12.*XNI*EX*EY*XLAMD*YLAMD/(XH2*DALFA)
      Q11=XNI*EX*XLAMD/DALFA
      Q22=XNI*EY*YLAMD/DALFA
      Q12=-0.5*((1.+YLAMD)*EX*XLAMD+(1.+XLAMD)*EY*YLAMD)/DALFA
      D11=(1.+XLAMD)/(DALFA*EXXP)
      D22=(1.+YLAMD)/(DALFA*EXXP)
      D12=((1.+XLAMD)*(1.+YLAMD)-XNI)/(DALFA*EXXP*(1.-XNI))
      DL1=DD*H11
      DL2=DD*XNI*(1.+(EX*EY*XLAMD*YLAMD*12.)/(XH2*DALFA))
      DL3=EX*XLAMD*(1.+YLAMD)/DALFA
      DL4=-XNI*EX*XLAMD/DALFA
      DL5=DD*H22
      DA1=(1.+YLAMD)/(DALFA*EXXP)
      DA2=-XNI/(DALFA*EXXP)
      DA3=-(1.+YLAMD)*EX*XLAMD/DALFA
      DA4=XNI*EY*YLAMD/DALFA
      DB2=(1.+XLAMD)/(DALFA*EXXP)
      DB3=XNI*EX*XLAMD/DALFA
      DB4=-(1.+XLAMD)*EY*YLAMD/DALFA
      MI(1)=2*K6
       MI(NEQPOT)=2*K6
      NEQ1=NEQPOT-1
      DO 10 I1=2,NEQ1
      MI(I1)=K6
   10 CONTINUE
      DELTA=XL/(NEQPOT-1)
      AL1=-1./(2.*DELTA)
      GA1=1./(2.*DELTA)
      AL2=1./(DELTA**2)
      BT2=-2./DELTA**2
      GA2=1./DELTA**2
      RETURN
      END
```

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```
      SUBROUTINE COEFNN(NNN)
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
       COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      C9=(NNN/RR)**2
      C1=DD*H11
      C2=2.*DD*H12*C9
      C3=2.*Q12*C9
      C4=DD*H22*C9**2
      C5=Q11/D11*C9
      C6=1./(RR*D11)*C9
      C7=C9**2/(2.*D11)
      C8=Q22*C9**2
      C10=C9/2.
      C11=2.*C9*D12
      C12=D22*C9**2
      RETURN
      END
```

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